Complex analysis questions for Comprehensive Exam in Analysis
September 2018

1. (a) Does every holomorphic mapping of the open unit disk D to itself have a fixed point? Why or why not?

   (b) Does there exist a holomorphic mapping of D *onto* $\mathbb{C}$? Explain.

2. Suppose that $f(z)$ is meromorphic in an open subset Ω of $\mathbb{C}$, and that $K \subset \Omega$ is a compact set with oriented boundary Γ. Assume that $f(z)$ does not take the value a on Γ and has no poles on Γ. Use the residue theorem to determine what is counted by the integral

$$\frac{1}{2\pi i} \int_\Gamma \frac{zf'(z)}{f(z) - a} dz \, .$$

3. Let $\{f_n\}$ denote a sequence of holomorphic functions on a domain $\Omega \subset \mathbb{C}$. Suppose that $\{f_n\}$ is a normal family, and that every subsequence $\{f_{n_k}\}$ which converges uniformly on compact sets converges to the *same* holomorphic function f on Ω. Prove that $\{f_n\}$ converges to f uniformly on compact subsets of Ω.