Abstract Algebra

Quest University Canada Block 1, Spring 2016

> *The notion of a "group," viewed only 30 years ago as the epitome of sophistication, is today one of the mathematical concepts most widely used in physics, chemistry, biochemistry, and mathematics itself.* – Alexey Sosinsky, 1991

Abstract algebra is the field of mathematics that studies algebraic structures such as groups, rings, fields, vector spaces and modules; we will primarily study groups in this course. The power of abstract algebra is embedded in its name: it gives us an arena in which we may study disparate mathematical objects together and abstractly, without considering a particular instance or occurrence. For example, the multiplication of numbers, symmetries of a molecule, dance formations, roots of polynomials, Australian kin systems, actions of a Rubik's cube, and loops on surfaces all form groups. By exploring groups abstractly, we can derive properties and structures that apply to all examples that we currently know or may discover in the future. With this in mind it should come as no surprise that abstract algebra builds a language that is used in nearly every field of mathematics.

The applications of the field within and beyond mathematics are not the only reasons to study abstract algebra. First, learning abstract algebra is one of the best ways to practice working through complex concepts and to develop your abstract reasoning abilities. Second, studying abstract algebra provides a window into what it is like to do research mathematics. Perhaps most importantly, you will experience the intrinsic beauty of mathematics during this course. While the aesthetic nature of abstract algebra is difficult to describe, it is obvious to any of its practitioners.

By the end of this course you will be able to:

1. Construct and understand properties of groups
2. Proficiently move between abstract groups and concrete examples
3. Understand mathematical structures deeply
4. Verify proofs and understand their value
5. Write proofs clearly and with proficiency
6. Present and defend mathematical arguments
7. Think abstractly and appreciate the need for abstraction
8. Communicate mathematics with greater clarity and more confidence

Course Information

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Course Topics

In this class we will primarily study group theory - the mathematics of symmetry. We will answer the following questions.

- **The Definition of a Group:** Where do groups arise? How can we visualize groups? How are groups described algebraically? What do we mean by the 'structure' of a group?
- **Examples of Groups:** What groups describe the symmetries of regular polygons and rotating objects? What is the structure of rearranging objects? In what sense are two groups the same? How do all groups arise out of permutations?
- **The Structure of Groups:** What happens when we collapse a group to a subgroup? How can we glue multiple groups together? What are maps between groups? What restrictions on the structure of a group are imposed by the number of elements that it contains?
- **Applications of Groups:** Is there a quintic formula analogous to the quadratic formula? What do groups tell us about the structure of molecules, games, and kin structures?
- **Larger Mathematical Questions:** What is the difference between equivalent and identical? What is a mathematical structure? How can we simultaneously consider two different structures? What is the difference between a formal proof and an argument? What goes into the construction of a proof?

Course Text

The required text for this class is *Contemporary Abstract Algebra*, 8th ed. by Joseph Gallian, available at the university's bookstore. You may also use the sixth or seventh edition.

Weekly Meeting

I will schedule at least one individual meeting with you each week to discuss your progress in the class. Please bring your assignments and any questions or concerns. If we run out of time during the meeting, please schedule a time to continue our discussion.

Our Approach: Inquiry-Based Learning

The only way to learn mathematics is to do mathematics. – Paul Halmos

We will follow an approach called *inquiry-based learning*, or IBL, in this class. While IBL is often seen as radical and uncomfortable by students encountering it for the first time, it should not be a great departure from what you have seen in your other classes at Quest. Here are some characteristics of an IBL classroom:

- Students look to each other (rather than to the instructor) to validate the ideas presented

- Students are responsible for acquiring their own knowledge
- Problems drive the material and form the basis for discussion
- Students critique the work of others and ask each other for clarification when necessary

IBL is known as a *student-centered* approach, meaning that you will hold most of the responsibility for your learning and understanding. This approach fits particularly nicely in an abstract algebra class, where the greatest challenge is simply understanding the definitions, lemmas, theorems, problems, and proofs that you encounter.

How will we implement the IBL approach this block? Every day, you will complete a reading and approximately 10-15 problems in the afternoon and evening. A significant portion of class the following day will be based around this work, and we will discuss the readings and dissect solutions to the problems.

Students learn abstract algebra at a largely individual pace. The same concepts that seem very natural to you might be quite difficult for another student to understand. Therefore, you should be prepared for the workload to vary from day-to-day throughout the block.

Course Requirements

> *If I have made any valuable discoveries, it has been owing more to patient attention than to any other talents* - Sir Isaac Newton

To pass this course you will need to do the following.

- Complete daily assignments, consisting of reading and problems
- Present solutions to problems in class
- Perform well on daily quizzes
- Be an active classroom participant, and contribute to the Group Zoo
- Complete weekly synthesis assignments

Daily Assignments

> *You have to be confused before you can reach a new level of understanding anything.*
> – Dudley Herschbach

Every day, you will be assigned a reading and a list of problems to complete. The reading will usually consist of one chapter from the textbook. The problems will serve several purposes and be of varying difficulty: some will be easy warm-ups to the reading or simply check details of presented proofs, while others will ask you to write intricate proofs. I expect that you will spend nearly all of your study time on these daily assignments.

Your notes on the assigned problems should contain enough detail to be easily understood by you at a later time. Results in abstract algebra build on each other, so you will refer to your solutions later in the block. Further, your notes will help you to swiftly present and discuss solutions during class. While you do not need to write your solutions in complete

sentences, they should be easy to follow and clear. If you decide to add to these notes during class before they are collected, you must use a separate colour to clearly distinguish what you prepared before class.

I will collect, grade, and return these assignments each day. You will receive one point (a ✓) for a question when it appears that you have put adequate thought and work into it (not necessarily if it is right or wrong). On most days I will just glance at your solutions, but I will occasionally read them in detail to ensure that you are actually working through the problem and not just writing lots of impressive-looking words.

Presenting Solutions[1]

> *The intelligence is proved not by ease of learning, but by understanding what we learn.*
> – Joseph Whitney

We have a lot of work to accomplish this block and your contributions in-class are crucial to the success of others. Student presentations of homework problems will be the centrepiece of the class, so you should take them seriously. Here are some things to know about making presentations in abstract algebra.

- The purpose of class presentations is not to prove to the instructor that you have done the problem. It is to make the ideas of the proof clear to the other students.
- You must use proper English and mathematical grammar during presentations
- Presenters should explain their reasoning as they go along, not simply write everything down and then turn to explain.
- Fellow students are allowed to ask questions at any point and it is the responsibility of the person making the presentation to answer those questions to the best of his or her ability.
- Presentations are directed at students, so the presenter should look at students to be able to see how well they are following the presentation.

I will seek volunteers to present solutions in-class, but you should expect to present solutions to problems every day. The following scale will be used to grade your presentations.

Mark	**Criteria**
4	Completely correct and clear proof or solution
3	Solution has minor technical flaws, unclear language, or lacking details. Essentially correct.
2	A partial explanation or proof is provided but a significant gap still exists
1	Minimal progress has been made, including relevant information and could lead to a proof
0	Completely unprepared

[1] from Carol Schumacher

Participation

The purpose of a proof is to understand, not to verify. – Paul Halmos

To learn abstract algebra and to contribute to the learning of your peers, you must be active and engaged during class. During a presentation you are not off the hook just because someone else volunteered. Even if you are sitting down, you are responsible for contributing questions and comments to help clarify what is being presented.

It is important to note that you should be just as eager to offer questions about concepts that you understand as those that you do not understand. For example, if you have done the problem and understand it well, you can often make suggestions that help clarify the wording or structure of a presentation.

During the course, you will be establishing a 'Group Zoo' on a wall in the classroom, to act as a public dictionary of examples of groups. When thinking about what to include in the group zoo, consider the following:

- Title of the entry: What is the symbol for the group?
- What is the group's Cayley table and Cayley diagram? If you cannot draw these easily, can you say anything about them? I have some nice computer generated pictures for most groups - if you would like access to the files, please ask!
- How are different groups linked? If you can establish links to different groups you should.
- Are there any groups with the same structure as this group?
- What examples do we see in this group?
- What can you say about the structure of the group?
- Are there any other cool facts that set this group apart? For example, is it the only group of a certain order? Does it have historical significance?

I expect that you will add to and reorganize the group zoo as the block goes on and you learn more about these 'animals'.[2]

Daily Quizzes

Every day, I will pose 1 to 4 basic questions about the material covered in the class so far. You may be asked to give a definition, a theorem, a proof, or an example. The purpose of these quizzes is to motivate you to learn material, and to provide you the opportunity to retrieve your knowledge in a variety of settings.

While the quiz questions will generally be shorter and less complicated than the ones that you present formally, the same 4-point grading scale will be used to grade these problems.

[2]The title 'group zoo' was inspired by my first abstract algebra professor, Dr. Pollak, who always referred to objects in algebra as 'animals.'

Synthesis Assignments

Each week, you will complete a written synthesis assignment consisting of full solutions to approximately five problems. Most of these problems will be discussed in class prior to the assignment due date. Since some of the problems have already been discussed, I will be paying significant attention to the style and completeness of your writing. The main purpose of these assignments is to learn how to write formal mathematical proofs like those you would see in a textbook or in a research paper. This means that *every* detail counts and needs to be included. See the handout 'How to write mathematics' for more information.

During the first week of class, we will develop a rubric which I will use to evaluate your mathematical writing throughout the course.

Academic Integrity

While googling a homework problem or trading solutions with a classmate may seem like good strategies for doing well in this class, these actions will prevent you from learning material, refining your problem-solving skills, and developing self-sufficiency and self-esteem.

The consequences for cheating are severe. *Any* blatant academic dishonesty will result in failure of the course and immediate reporting to the Chief Academic Officer.

Discussing problems with classmates and building off of each others' ideas is not cheating. However, the following following actions *are* considered cheating.

- Reading about problems on the internet or in other abstract algebra textbooks. You need to struggle through the problems and ideas of this course as independently as possible!
- Copying the writing or explanations of mathematical work from someone else
- Looking at a solutions manual for the textbook or any solutions online, at any time (even for daily homework questions)

These examples are not comprehensive; if you have questions about whether something is considered cheating, please speak with me first.

Grading

Your final grade will be calculated as follows.

25%	Daily Assignments
25%	In-Class Presentations and Participation
5%	Group Zoo (class grade)
20%	Daily Quizzes
25%	Synthesis Assignments

The course grading scale is:

A	93-100%	B	83-86%	C	73-76%
A-	90-92%	B-	80-82%	C-	70-72%
B+	87-89%	C+	77-79%	D	60-69%

Narrative Evaluation

Any student at Quest can request a narrative evaluation (e.g. a written paragraph) in addition to their letter grade in any course. A narrative evaluation will give you more comprehensive feedback that you can learn from and additional information to present employers and graduate schools. If you wish to take advantage of this option, you have until the end of the 6th day of a course to sign up on the Registrar's Office Portal site.

Disability Accommodations

If you have a disability for which you seek accommodation, please make sure to have registered with the Learning Commons, as specified in the Student Accommodation Policy (`http://www.questu.ca/pdfs/_uploads/content/student_accommodation_policy.pdf`), and provide us with your Memorandum by the second day of class.

Additional Course Policies

- Please be on time to class. If you are consistently late you will be asked to leave.
- I expect that you will attend every class session. If you must miss a class for a valid reason (such as illness or a family emergency), please let me know *before* class. I reserve the right to ask for documentation to excuse your absence. For every class that you miss without a valid reason, 5% will be deducted from your *full* course grade.
- Bring pencils, paper, scientific calculator, and laptop to every class.
- Keep your cell phone in your bag during class. If you using or looking at your cell phone, you will be asked to leave.
- Always be respectful in your speaking and actions. Do not use profanity.
- All homework is due at the beginning of class, and all deadlines are absolutely firm. I will not accept late homework since we need to be able to discuss solutions in class, and because staying on top of deadlines encourages you to keep up with course material.
- Office hours: If you are not able to make my drop-in office hours, or need to speak with my privately, please e-mail me to arrange an appointment.
- E-mail: During the block I check my e-mail on weekdays around 9 AM and 6PM, and sporadically throughout the rest of the days and weekends. Please do not e-mail me with questions that may be easily answered by looking at this syllabus, the course website, or asking other members of the class. Be polite and use proper English grammar.
- Please do not bring food into the classroom. You may bring drinks.

Schedule

Roughly, we will be focusing on the following topics from week to week. See the 'Course topics' section for more details on these topics.

- **Week 1**: The Definition of Groups
- **Week 2:** Examples of groups
- **Week 3:** The Structure of Groups
- **Week 4:** Galois Theory and Unsolvability

The table below summarizes the key deadlines.

Date	Assignment Due
Monday, January 18	Synthesis Assignment 1
Monday, January 25	Synthesis Assignment 2
Monday, February 1	Synthesis Assignment 3
Wednesday, February 3	Synthesis Assignment 4