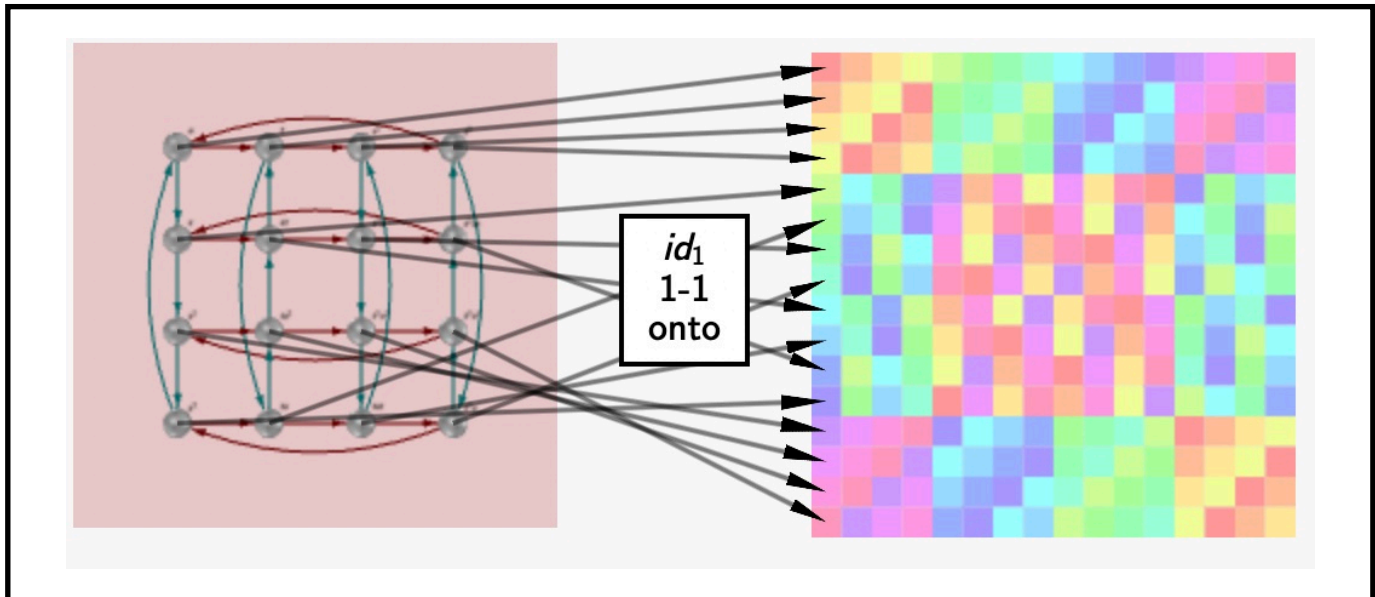


GROUPS & SYMMETRIES

UNIVERSITY OF TORONTO, WINTER 2022 THE



WHAT ARE GROUPS?

The notion of a "group," viewed only 30 years ago as the epitome of sophistication, is today one of the mathematical concepts most widely used in physics, chemistry, biochemistry, and mathematics itself. - Alexey Sosinsky, 1991

Abstract algebra is the field of mathematics that studies algebraic structures such as groups, rings, fields, vector spaces and modules; we will primarily study groups in this course. The power of abstract algebra is embedded in its name: it gives us an arena in which we may study disparate mathematical objects together and abstractly, without considering a particular instance or occurrence. For example, the multiplication of numbers, symmetries of a molecule, dance formations, roots of polynomials, Australian kin systems, actions of a Rubik's cube, and loops on surfaces all form groups. By exploring groups abstractly, we can derive properties and structures that apply to all examples that we currently know or may discover in the future.

With this in mind it should come as no surprise that abstract algebra builds a language that is used in nearly every field of mathematics.

Course Delivery:
synchronous online & in person(?)

Webpage:
q.utoronto.ca/courses254640

Public Questions:
Piazza message board

Contact for Private Questions:
smt@math.toronto.edu

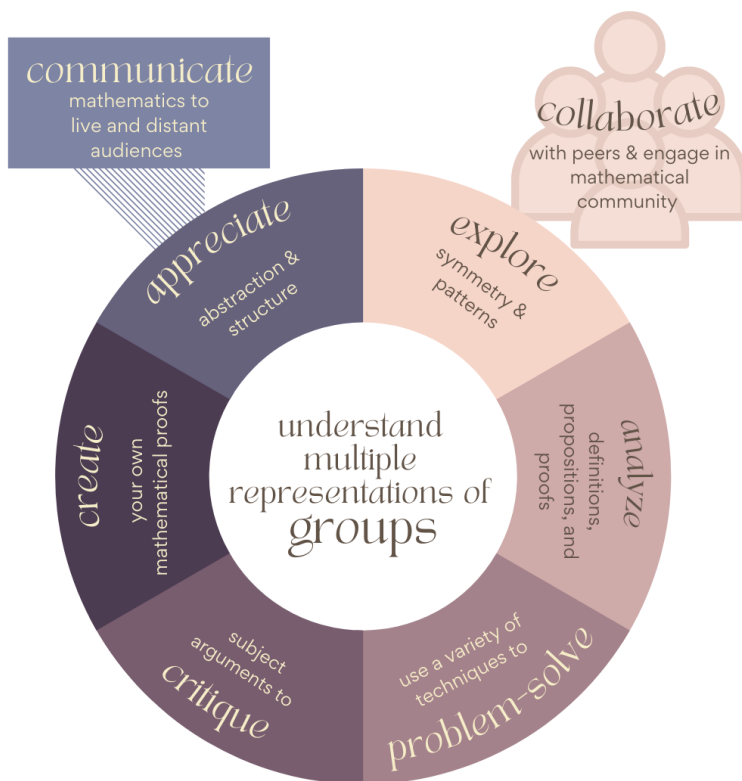
Office Hours: Zoom drop-in with professor & TAs;
Schedule posted online

Text: Contemporary Abstract Algebra by Gallian (you must purchase in bookstore to have access via Perusal!!)

SKILLS

To successfully complete this course, students will **deeply understand multiple representations of groups** and **flexibly use these understandings**. To develop and support these understandings, by the end of the course, they should:

1. Appreciate abstraction and structure
2. Analyze and select appropriate mathematical definitions, propositions, and theorems .
3. Communicate complex mathematics using clear and customary mathematical language and notation appropriate for a given audience, both orally and in writing.
4. Demonstrate proving and problem-solving techniques including generating examples, pattern exploration, posing conjectures, creative proof construction, and generalization of results.
5. Collaborate with peers, and engage ethically in mathematical community
6. Explore symmetry in abstract and real-life contexts
7. Create their own mathematical proofs
8. Critique mathematical arguments



WHO AM I?

I am Professor Sarah Mayes-Tang. I've been a Teaching-Stream Professor at the University for about 5 years, and prior to that I was a professor at Quest University Canada for four years. I earned a PhD in mathematics from the University of Michigan with a dissertation entitled *Asymptotic Generic Initial Systems*, and a bachelor's degree in pure math from Queen's University. As a Teaching Stream Professor I am interested in questions related to how students and teachers learn and how classes can run more effectively.

I - AND THE REST OF THE TEACHING TEAM - TRUST THAT YOU ARE DOING YOUR BEST. YOUR BEST RIGHT NOW MAY NOT BE WHAT IT WOULD BE UNDER "NORMAL" CIRCUMSTANCES. AND THAT'S OKAY.

YOUR WORTH IS NOT DETERMINED BY A GRADE OR A COURSE OR EVEN A DEGREE. YOU ARE WORTH JUST AS MUCH CARE AND TIME AND ENERGY AS ANYONE ELSE. I AIM TO CREATE A CLASSROOM ENVIRONMENT WHERE YOU FEEL THAT WORTH FROM EVERYONE.

- from the *Impact and Worth* Statement in the full syllabus

KNOWLEDGE

A sample of the questions we will tackle are: We will answer the following questions.

- **The Definition of a Group:** Where do groups arise? How can we visualize groups? How are groups described algebraically? What do we mean by the 'structure' of a group?
- **Examples of Groups:** What groups describe the symmetries of regular polygons and rotating objects? What is the structure of rearranging objects? In what sense are two groups the same? How do all groups arise out of permutations?
- **The Structure of Groups:** What happens when we collapse a group to a subgroup? How can we glue multiple groups together? What are maps between groups? What restrictions on the structure of a group are imposed by the number of elements that it contains?
- **Applications of Groups:** Is there a quintic formula analogous to the quadratic formula? What do groups tell us about the structure of molecules, games, and kin structures? How many wallpaper patterns are there?
- **Larger Mathematical Questions:** What is the difference between equivalent and identical? What is a mathematical structure? How can we simultaneously consider two different structures? What is the difference between a formal proof and an argument? What goes into the construction of a proof?

The group theory that forms the basis of your learning can be found in from:

- The primary course text, *Contemporary Abstract Algebra*, by Gallian (Chapters 0-11, plus Sylow's Theorems)
- Dana Ernst's text, *An Inquiry-Oriented Approach to Abstract Algebra* (Chapters 1-9, plus the appendices), available [here](#)
- Examples of groups that you generate in Group Explorer
- Other supplementary texts.



(Note all dates are "Toronto time")

Preps: Due each Friday @ 9 am

Base Camps: Due each Friday @ 5 pm

Problems: Ready for your 1-hour class on Wed or Thurs

Midterm: March 9-10 (in-class)

Exploration Deadlines:

Feb 3 at 8 pm

March 10 at 8pm

April 7 at 8pm

ASSIGNMENTS

- **Prep Assignments** that ask you to read, watch, and / or listen to material and then discuss it with your classmates. These will take place on Perusall. There will be 12 of these, due each Friday at 9 am.
- **Base Camp Assignments** that are *part of* what you will do in tutorial. These will ask you to collaborate with others via a discussion thread as you find errors in proofs, analyze theorems, or unpack definitions. I anticipate that most will take place on Perusall. There will be 9 of these, due on Base Camp tutorial weeks at 5pm.
- **Exploration Assignments** that will allow you to apply previous material to contexts outside of the textbook and anticipate material that is coming up next. You will move quickly from research through final draft during the final week of each unit. There will be 3 Exploration Assignments.
- Your **Group Journal** is more of a record of your group's work than an assessment. It will be a compilation of the proofs that you agree on as a group, and any significant discussions about them that you have. The scribes each week are responsible for making sure that the journal is up-to-date.
- **Quizzes** in this course will aim to draw in large part from the work in Group Journals. I will present an excerpt from a group's Journal (not necessarily your own journal) and ask you to critique it in some specific way (e.g. identify an error, find something done well, explain what they are doing). There will be 3 quizzes in total, one for each unit.
- **The Test** will test group theory knowledge, logic, and problem-solving skills. Stage 1 of test will be individual, and Stage 2 of the test will be with your Bourbaki Groups. Stage 2 will be (mostly) the same as Stage 1, with the potential of some extra challenges added.
- **The Final Exam** will be an oral exam. Your Group Journal will be the only aid permitted. More details will be given closer to the date.

OVERALL COURSE GRADES

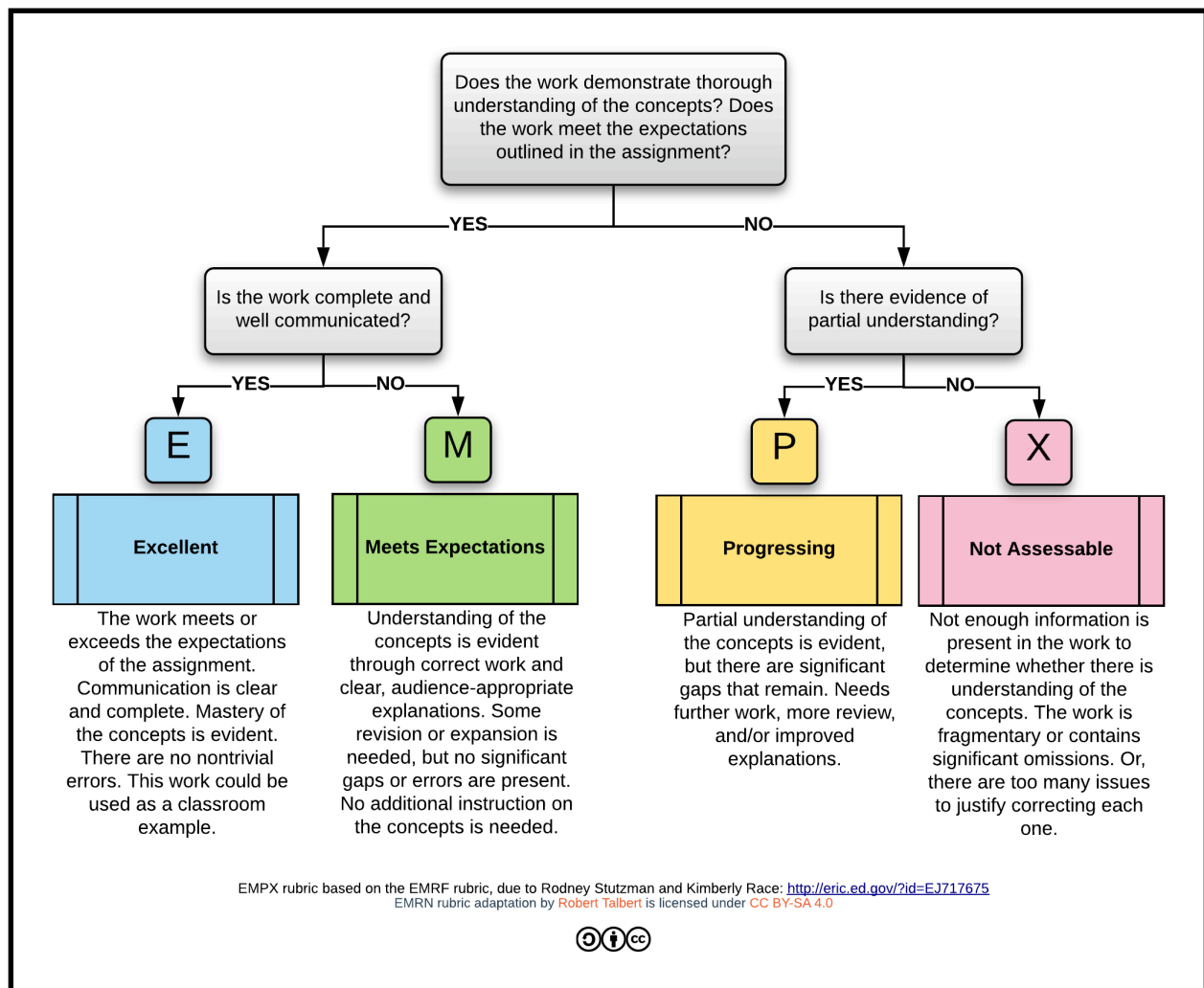
Your grade in the course is earned by **demonstrating evidence of skill on the main concepts in the course** and by **showing appropriate engagement with the course**. And this is done by completing the assignments outlined above, at a reasonably high level of quality.

In this class, **there are no points or percentages** on any items. Instead, your work will be evaluated against **quality standards** that will be made clear on each assignment. If your work meets the standard, then you will receive full credit for it. Otherwise, you will get feedback and, when it makes sense, the chance to reflect on the feedback, revise your work, and then resubmit it for regrading.

The individual assignments are graded as shown in the table on the next page.

How individual assignments are marked:

Assignment	How it's marked
Prep Assignments	below expectations (0), meets expectations (1), above expectations (2)
Base Camps	below expectations (0), meets expectations (1), above expectations (2)
Quizzes & Surveys	Complete (if score above 75%), Incomplete (if score at or below 75%)
Group Notebook & Group Contributions	Pass or No Pass
Explorations	EMPX (see below)
Midterm & Final	EMPX (see below)



grade table

Grade	Preps (at least "meets expectations")	Base Camps (at least "meets expectations")	Quizzes & Surveys	Group Notebooks & Group Contributions	Explorations (at least "M")	Midterm & Final (at least "M")
A	10 (out of 12)	10 (out of 12)	3 (out of 3) quizzes; all surveys	pass on both	1 E; 2 M or E	E on one / M on other
B	9	9	3	pass on both	2	M on both
C	8	8	2	pass on one	2	M on both
D	4	4	1	n/a	1	M on one

Your final grade in the course is determined by the table. Each grade has a *requirement* specified in its row in the table. **To earn a grade, you will need to meet all the requirements in the row for that grade.** Put differently, your grade is the **highest** grade level for which **all** the requirements in a row of the table have been met or exceeded.

A grade of F is given if **none** of the rows has been fully completed.

Plus/minus grades: Plus/minus grades will be assigned at my discretion based on how close you are to the next higher or lower grade level.

GRADING WORK

- **Prep assignments and most Base Camp Assignments:** Automatically graded using Perusall.
- **Quizzes:** Consists of multiple choice questions, be graded as a percentage.
- **Surveys:** Completion noted
- **Group Notebook:** Must (a) meet standards and (b) be complete to achieve a "pass"
- **Group Contributions:** Must (a) attend 10/12 meetings and (b) receive passing peer evaluations to achieve a "pass"
- **Explorations:** Criteria will be specified on the individual assignments; they will use the EMPX standards above.
- **Midterm & Final:** Criteria will be specified when the specific format of the assessments is published.

TOKENS

Each student has 5 tokens. They can be redeemed for items such as:

- Extend any deadline *except for Prep or Base Camp Assignment* by 24 hours
 - Convert a "Does not meet expectations" on a Prep Assignment to a "Meets expectations".
 - Convert a "Does not meet expectations" on a Base Camp assignment to a "Meets expectations".
 - Convert a "nonattendance" in a group meeting to an "attendance".
 - Revise an Exploration marked "X".
- See online Syllabus for more information on Tokens and on resubmitting.

TOOLS

Tools you will need for this course:

- Computer & Internet access satisfying the University's minimal technical requirements.
- Latest version of Zoom
- Working microphone that you can use *at least* during tutorial and the 1-hour class each week
- A way to take pictures of your work (e.g. a phone camera)
- A tablet will be very helpful (you don't need an iPad! Get an inexpensive one)
- Course textbook

COURSE MATERIALS & VIDEOS ARE COPY-RIGHTED

Do not share any note, video, or anything else related to the course without the written permission of the Professor. This include close family members and employes. (See the syllabus for the full copy-right statement)

EQUITY: A COURSE VALUE

As we all adjust to the changes that COVID-19 has brought, and increasingly participate in virtual learning environments, students are reminded of the expectation that we all demonstrate respect for one another. As outlined in the Student Code of Conduct, the University of Toronto does not condone discrimination or harassment against any persons or communities especially when based on grounds protected under the Ontario Human Rights Code. [The University of Toronto's Equity Offices](#) remain available to students to provide support on equity issues that arise as a result of COVID-19.

Students are encouraged to support one another and the University's commitment to human rights and our values of diversity, inclusion, and respect in managing any inappropriate comments or disruptive behaviours. If you experience or witness inappropriate comments or behaviours in your classes, you are encouraged to contact your instructor. If you can, take and share a screenshot of the inappropriate content with me - Prof. Mayes-Tang - so that I can follow-up with you and address the conduct.

Equity in Math

Math does not represent the demographics of the country or the world at large. Likewise, the math department and the math courses at UofT do not adequately represent the diversity of Toronto's population, in terms of race, gender, or other diversity metrics.

The actions that we take and the things that we say within our mathematical spaces - whether they be physical or virtual, or "official" or "informal" - create the culture of math at UofT. It is this culture that shapes people's experience with math at the University.

I treat the responsibility to shape the mathematical culture positively and openly very seriously. Think about the consequences that your words and actions have. For example, when you talk about someone being "good at math" or a "genius", what other stereotypes are you also calling to mind? If you say that someone is "surprisingly" good at math, what stereotypes are you invoking? Even harmless statements can have a big impact.

COURSE ACTIVITIES

There are three **synchronous** course activities each week of the course.

1. **Base Camps**, taking place during tutorials, will assist you in establishing your basic knowledge of group theory and the logical structure that underlies it. They will help firm up your understanding of the reading before attempting the problems for the week.
2. **The 2-hour lecture:** This takes place on Zoom for the first 3 weeks of class and in-person if / when we are allowed to go back to campus. It is an interactive time designed to make you *think*. I design classes first on the maxim that "the only way to learn math is to do math" and second on the principles of learning science.
3. **The 1-hour session** of the class will be dedicated to you talking about the week's problems in groups. Before this session you will have attempted all of the problems yourself (you may also work with others before this session, of course, and are encouraged to!). The purpose of this session is to collect ideas, critique each others' arguments, and build up your own groups' knowledge of the subject in a way that makes sense to you. Everything that you do and see in group theory should make sense to you - and these 1 hour "talks" sessions are intended to ensure that they do. You'll be given very specific scripts for the sessions to use.
4. **Exploration Weeks.** At the end of every 4-week unit, we will have an Exploration week, where tutorials and classes will look different than during the other 3 weeks: there will not be much new material, but we will be reviewing and applying what you learned.

The following image should give you an idea of the "routine" that we will follow.

Unit 1 Schedule

Mon	Tues	Wed	Thurs	Fri
2-hour class (1.1a)		1-hour: Group Talks (1.1a)		1.1b Prep Due / Base Camp Tutorial
2-hour class (1.1b)		1-hour: Group Talks (1.1b)		2.1a Prep Due / Base Camp Tutorial
2-hour class (1.2a)		1-hour: Group Talks (1.2a)		Explore Prep Due / Tutorial
2-hour class (Explore & Review)		1-hour: Group Review		2.1a Prep Due / Base Camp Tutorial

FULL COURSE CALENDAR

Unit	Chapters of Gallian	Dates	Activities
1.1: Definition of Groups	0-2	January 10-20	Preps 1.1a & 1.1b Base Camps 1.1a & 1.1b Problems 1.1
1.2: First Examples of Groups	3-4	January 21-February 3	Preps 1.2a & 1.2b Base Camp 1.2a Problems 1.2 Unit 1 Exploration
2.1: When Groups are the "Same"	5-7	February 4-17	Unit 1 Problems Quiz Preps 2.1a & 2.1b Base Camps 2.1a & 2.1b Problems 2.1 Group Peer Evaluation Check
2.2: Combining Copies of Groups	8	February 18-March 10	Preps 2.2a & 2.2b Base Camp 2.2a Problems 2.2 Unit 2 Exploration Mid-Course Test
3.1: Digging Deep into the Structure of Groups	9-11	March 11-24	Unit 2 Problems Quiz Preps 3.1a & 3.1b Base Camps 3.1a & 3.1b Problems 3.1
3.2: Using Sylow Theorems to Determine Group Possibilities	24-25	March 25-April 7	Preps 3.2a & 3.2b Base Camp 3.2a Problems 3.2 Unit 3 Exploration Unit 3 Problems Quiz

WHERE CAN I FIND SUPPORT?

If you want to know, you ask the question. There's no such thing as a dumb question. It's dumb if you don't ask it. --Katherine Johnson

Mental health support

- You may find yourself feeling overwhelmed, depressed, or anxious. Lots of people feel the same way. There is help available from mental health professionals 24 hours a day via online and phone-based services. Here are some that are available to U of T students:
 - [MySSP - My Student Support Program](#) 1-844-451-9700, or outside of Canada call 001-416-380-6578
 - [Good2Talk Student Helpline](#) 1-866-925-5454, or text GOOD2TALK to 686868
 - [Distress Centres of Greater Toronto](#) 416-408-4357, or text 45645
- There is also the new Navi tool for U of T students, it is a chatbot and your questions are totally anonymous. <http://uoft.me/navi> (Links to an external site.)



THE
STUDENT
UNION HAS
CURATED A
LIST OF
MENTAL
HEALTH
RESOURCES
HERE: [HTTPS://
WWW.UTSU.CA/
MENTAL-HEALTH/](https://www.utsu.ca/mental-health/)



A LIST OF UNIVERSITY
FINANCIAL SUPPORTS,
WORK-STUDY
OPPORTUNITIES, AND
GOVERNMENT
PROGRAMS IS AVAILABLE
ON THE
UNIVERSITY'S [FINANCIAL
SUPPORT & FUNDING
OPPORTUNITY
DIRECTORY](#).

THANK YOU TO MELISSA
CHEYNEY AT OREGON
STATE FOR THE WORDING
OF THE ABOVE.
THANKS TO PROF. LIZA
BOLTON FOR SHARING
THE LIST OF STUDENT
RESOURCES WITH ME!
THANK YOU TO ROBERT
TALBERT FOR SHARING

General University resources

- The following are some important links to help you with academic and/or technical service and support:
 - **Health & Wellness** [Links to an external site.](#)
can help with appointments with a range of clinicians, nutrition, immunizations, sexual and reproductive health and much more. Many of their services continue to be available online.
 - **Arts & Sciences** student resources through Sidney Smith Commons Online [Links to an external site.](#)
 - **General** student services and resources at [Student Lifeal site](#). Tips for dealing with multi-choice questions [Links to an external site.](#) (MCQs)
 - Book an appointment with a [learning strategist](#) [Links to an external site.](#)
(they can help you with strategies for MCQs also)

More University resources....

- Resources on **academic support** from the [Academic Success Centre](#) [Links to an external site.](#)
- Learner support at the [Writing Centre](#) [Links to an external site.](#)
- Information about **Accessibility Services** [Links to an external site.](#)
- Quercus Information in the [Canvas Student Guide](#) ([Links to an external site.](#))
- Logistical and social support for **international students** at the [Center for International Experience](#)

If you have further questions, please email ask.artsci@utoronto.ca.

Arts & Science COVID19 FAQ

The Arts & Science Undergraduate FAQ page addresses frequently asked questions that are specific to undergraduate students taking courses with the Faculty of Arts & Science. This page may be updated as more information comes available. We encourage you to check back often.

Messages from Dean Woodin and other news can be found on the [A&S latest updates page](#).

ACADEMIC ACCOMMODATIONS

I want to ensure that all students have the best possible learning experience in MAT301, and understand that equity is not the same as equality.

The University provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a collective obligation to develop an accessible learning environment that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs.

Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require accommodations, please feel free to approach me and/or the [Accessibility Services](#)

Links to an external site.

STUDENT PARENTS & CARERS

The University of Toronto strives to provide a family-friendly environment. You may wish to inform me if you are a student with family responsibilities. If you are a student parent or have family responsibilities, you also may wish to visit the Family Care Office website.

In addition, While we are online and school is not in session, I understand that childcare will be a major issue for student parents. Please do not let caring responsibilities get in the way of attending class. **All students should recognize that children are a part of our COVID-19 virtual "background noise"**. When we are in-person, children and babies are welcome in class under certain circumstances, See Online syllabus for more.

RELIGIOUS OBSERVANCES

The University provides reasonable accommodation of the needs of students who observe religious holy days other than those already accommodated by ordinary scheduling and statutory holidays. Students have a responsibility to alert members of the teaching staff in a timely fashion to upcoming religious observances and anticipated absences and instructors will make every reasonable effort to avoid scheduling tests, examinations or other compulsory activities at these times.

Please reach out to me **as early as possible** to communicate any anticipated absences related to religious observances, and to discuss any possible related implications for course

ACADEMIC INTEGRITY

This statement is long because I want to make it clear what is and what is not allowed in this course. You should still read on, but all of it should make common sense: anything that feels like a "short-cut" around doing the hard work of learning group theory yourself is wrong. I will not tolerate it. All members of the teaching team are on the look-out for signs that there may be an academic integrity violation.

Academic integrity is fundamental to learning and scholarship at the University of Toronto and beyond. Participating honestly, respectfully, responsibly, and fairly in this academic community ensures that the UofT degree that you earn will be valued as a true indication of your individual academic achievement, and will continue to receive the respect and recognition it deserves. Violating standards of academic integrity will prevent you from learning material, refining your problem-solving skills, and developing self-sufficiency and self-esteem.

The Prof Mayes-Tang and MAT301 TAs are strongly committed to assigning grades based on our students' honest efforts to demonstrate learning in this course. Academic dishonesty in any form will thus not be tolerated in this course. There are also practical reasons for you to follow academic integrity guidelines. If you do not have the skills that you are supposed to build in this course, you will not be prepared for the other courses in your degree program and will not be prepared for your job. Your future instructors and employers DO notice!

Students are expected to know what constitutes academic integrity: familiarize yourself with the information from the [Student Academic Integrity website](#) Links to an external site.

. It is the rule book for academic behaviour at the U of T. Two examples of academic offences are:

- Having another student write a homework assignment, test, exam, or impersonating someone else in writing one of these assessments
- Posting course materials (including quizzes, announcements, homework, tests, quizzes etc) online - in any language and in any country¹¹¹¹

The following actions are examples of things that are *not* offences in this class:

- Discussing questions from homework with classmates, building off of each others' ideas
- Collaborating on Base Camp assignments and on the reading problems

In accordance with the University's [Code of Behaviour on Academic Matters](#) , we will [actively investigate any suspected cheating, plagiarism, misrepresentation or other dishonest practices. The consequences for academic misconduct can be severe, including a failure in the course. If you have any questions about what is or is not permitted in this course, please do not hesitate to contact your instructor or TA.](#)

Students are usually reluctant to report incidents of academic Academic Integrity This statement is long because I want to make it clear what is and what is not allowed in this course. You should still read on, but all of it should make common sense: anything that feels like a "short-cut" around doing the hard work of learning group theory yourself is wrong. I will not tolerate it. All members of the teaching team are on the look-out for signs that there may be an academic integrity violation