Building Mathematical Understanding by Understanding Our Students

Sarah Mayes-Tang (University of Toronto)

CMS Winter Meeting

December 8, 2019

math

↓

career & study opportunities

math

parent & citizen

scientist & policy leader

mathematician & teacher

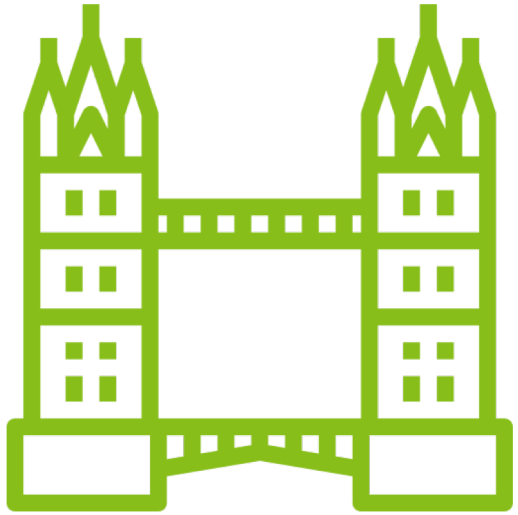
Our work as
math educators
builds the future of
mathematics

Our work as math educators builds the future of mathematics in

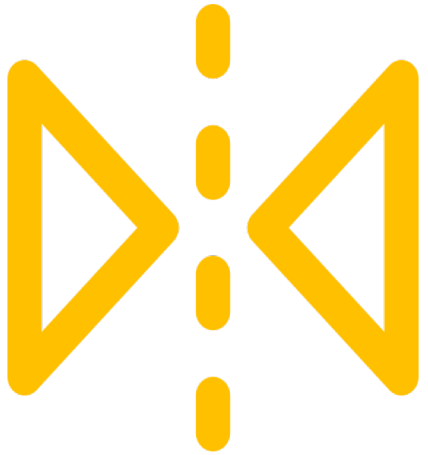
4 Ways to Build Mathematical Understanding

Develop a vision of & for your students

Design a story for your students



Build bridge for students' learning

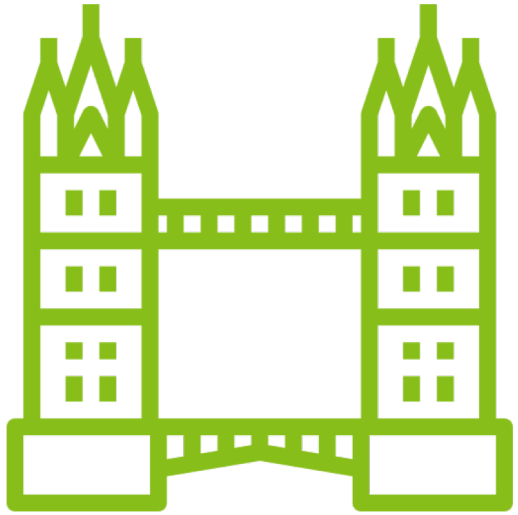


Advance & reflect

4 Ways to Build Mathematical Understanding & Diversity

Develop a vision of & for your students

Design a story for your students



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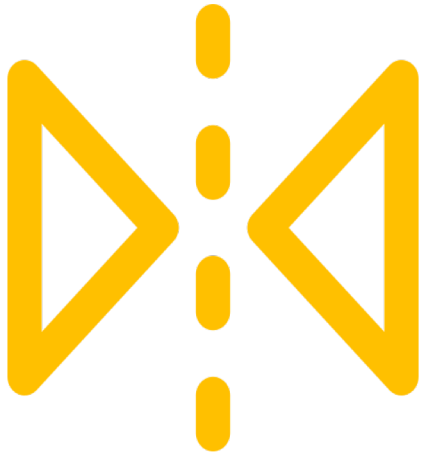


Advance & reflect

OPEN ACCESS PEER-REVIEWED

RESEARCH ARTICLE

Women 1.5 Times More Likely to Leave STEM Pipeline after Calculus Compared to Men: Lack of Mathematical Confidence a Potential Culprit

Jessica Ellis, Bailey K. Fosdick, Chris Rasmussen

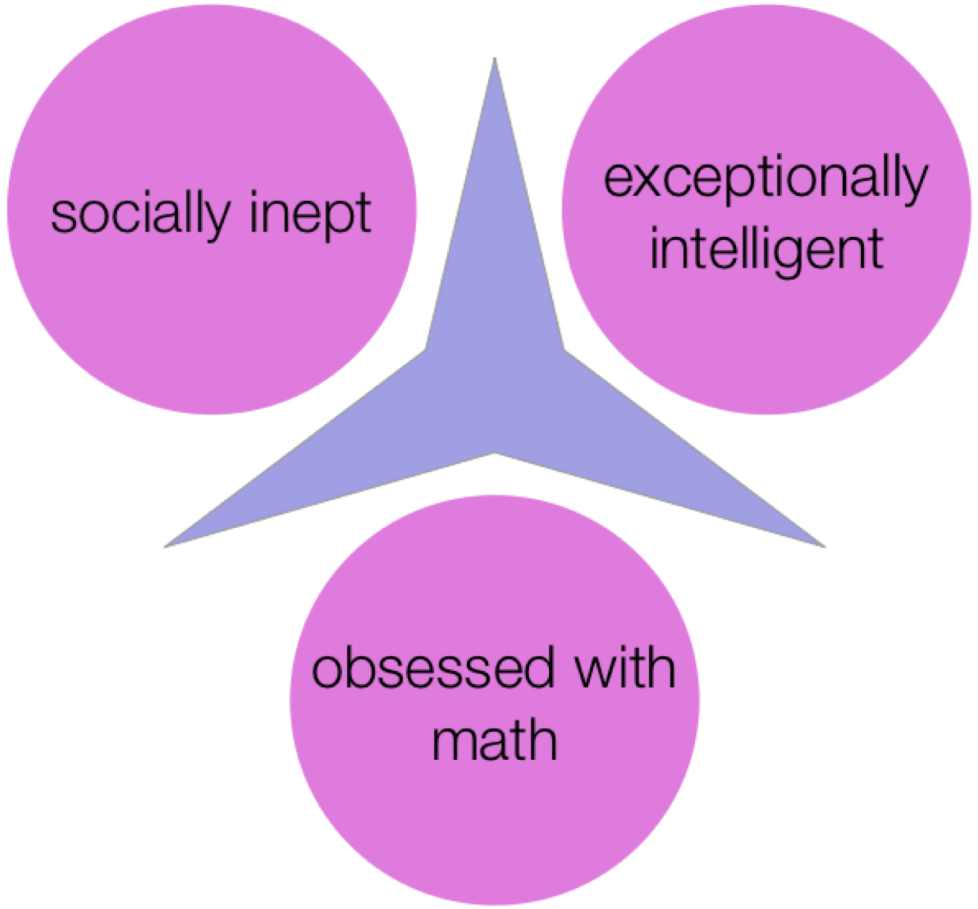
Published: July 13, 2016 • https://doi.org/10.1371/journal.pone.0157447

83 Save	44 Citation
26,223 View	288 Share

Low Confidence

- Factors that keep students from pursuing math are more likely to impact women & members of underrepresented groups

Ellis, J., Fosdick, B. K., & Rasmussen, C. (2016). Women 1.5 times more likely to leave STEM pipeline after calculus compared to men: Lack of mathematical confidence a potential culprit. *PloS one*, *11*(7), e0157447.



Perceptions of Mathematicians

- Factors that keep students from pursuing math are more likely to impact women & members of underrepresented groups

Piatek-Jimenez, Katrina. "Images of mathematicians: a new perspective on the shortage of women in mathematical careers." *ZDM* 40.4 (2008): 633-646.

"These women students' invisibility is not biddable. It is intentional. Their self-identification as "special" is not masculine. It is protective. And some are finding ways to participate."

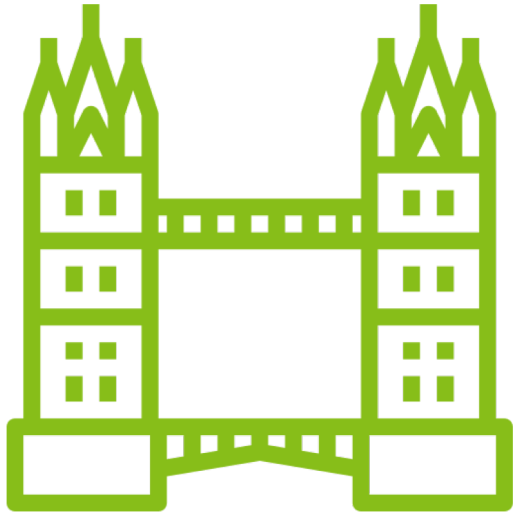
- Rodd & Bartholomew

Rodd, M., & Bartholomew, H. (2006). Invisible and special: Young women's experiences as undergraduate mathematics students. *Gender and Education*, *18*(1), 35-50.

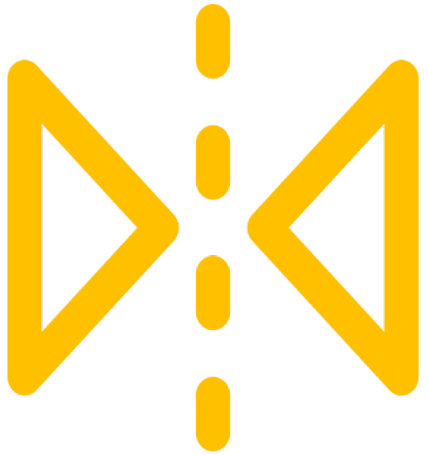
4 Ways to Build Mathematical Understanding & Diversity

Develop a vision of & for your students

Design a story for your students



Build bridge for students' learning



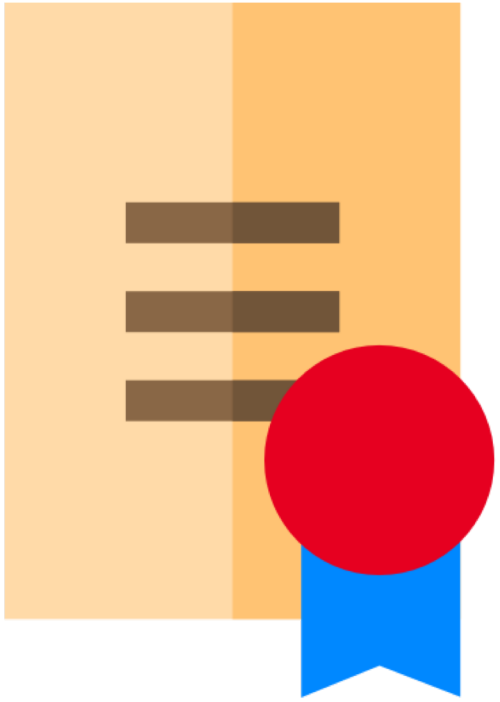
Advance & reflect

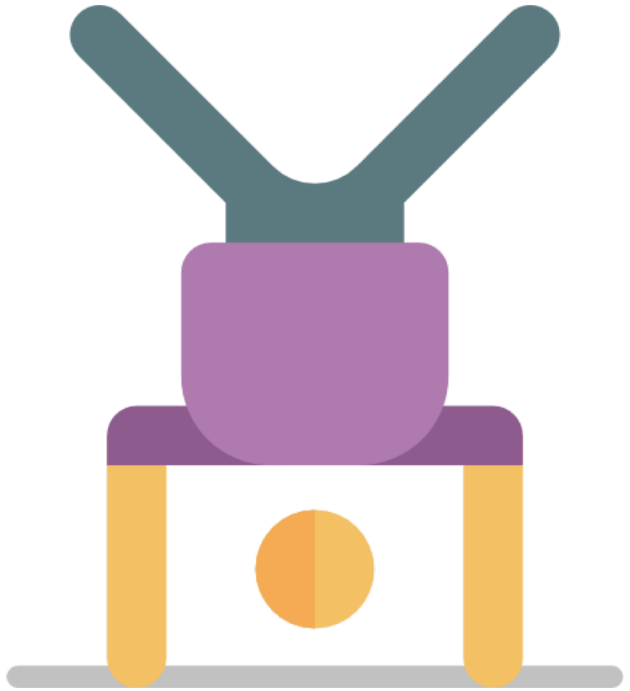
Develop a vision OF and FOR your students

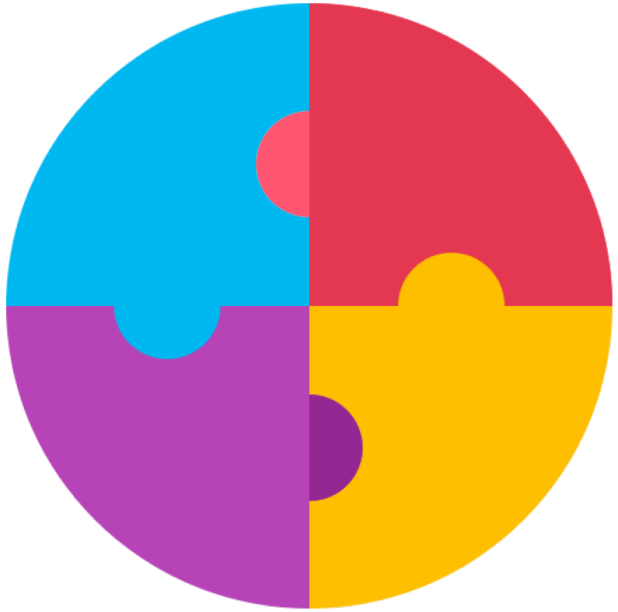
Who are your students today?

who students are impacts their learning

Exercise:
Make a mental portrait of a student in your course or program.

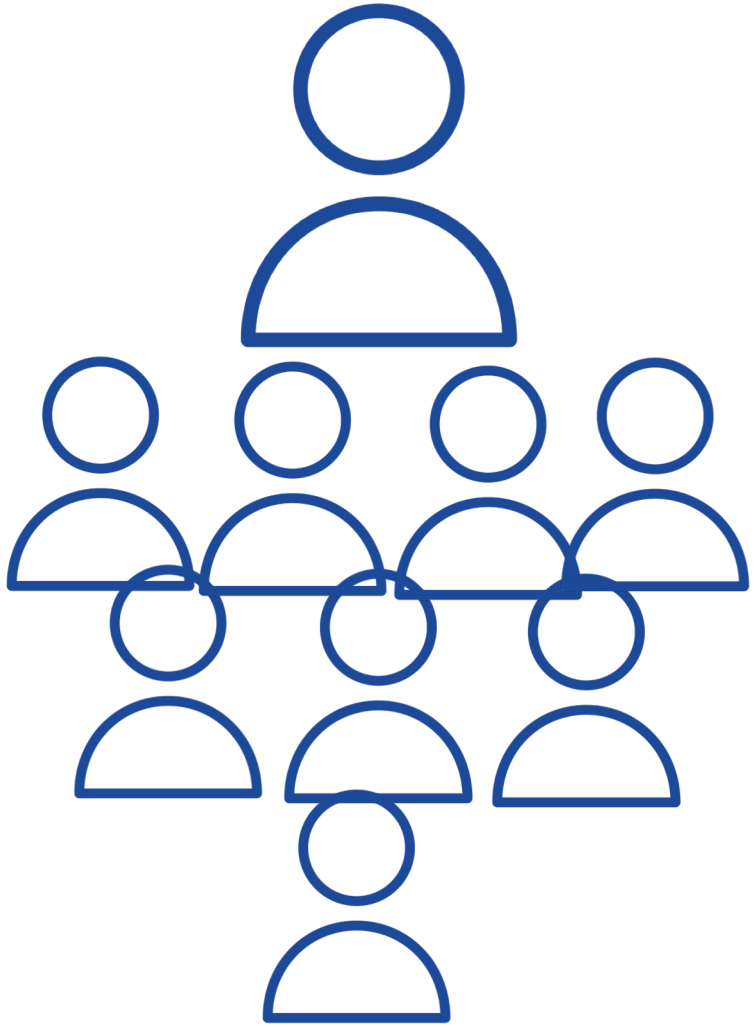




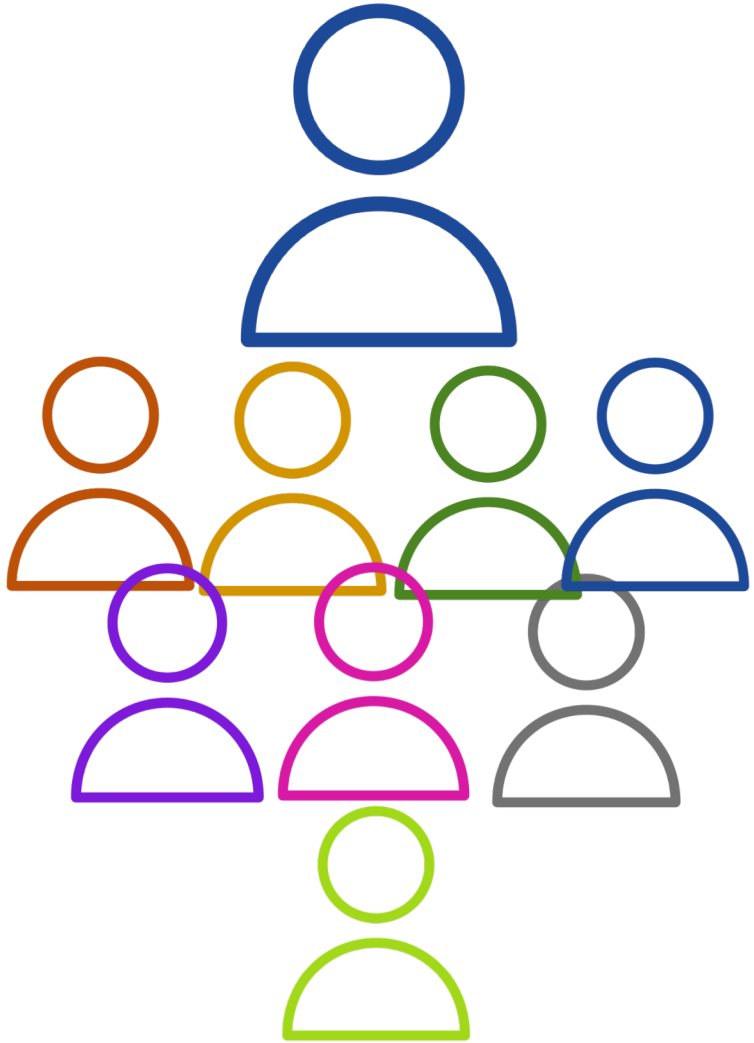


Example: MAT135/136, Introductory Calculus

false consensus effect: perception

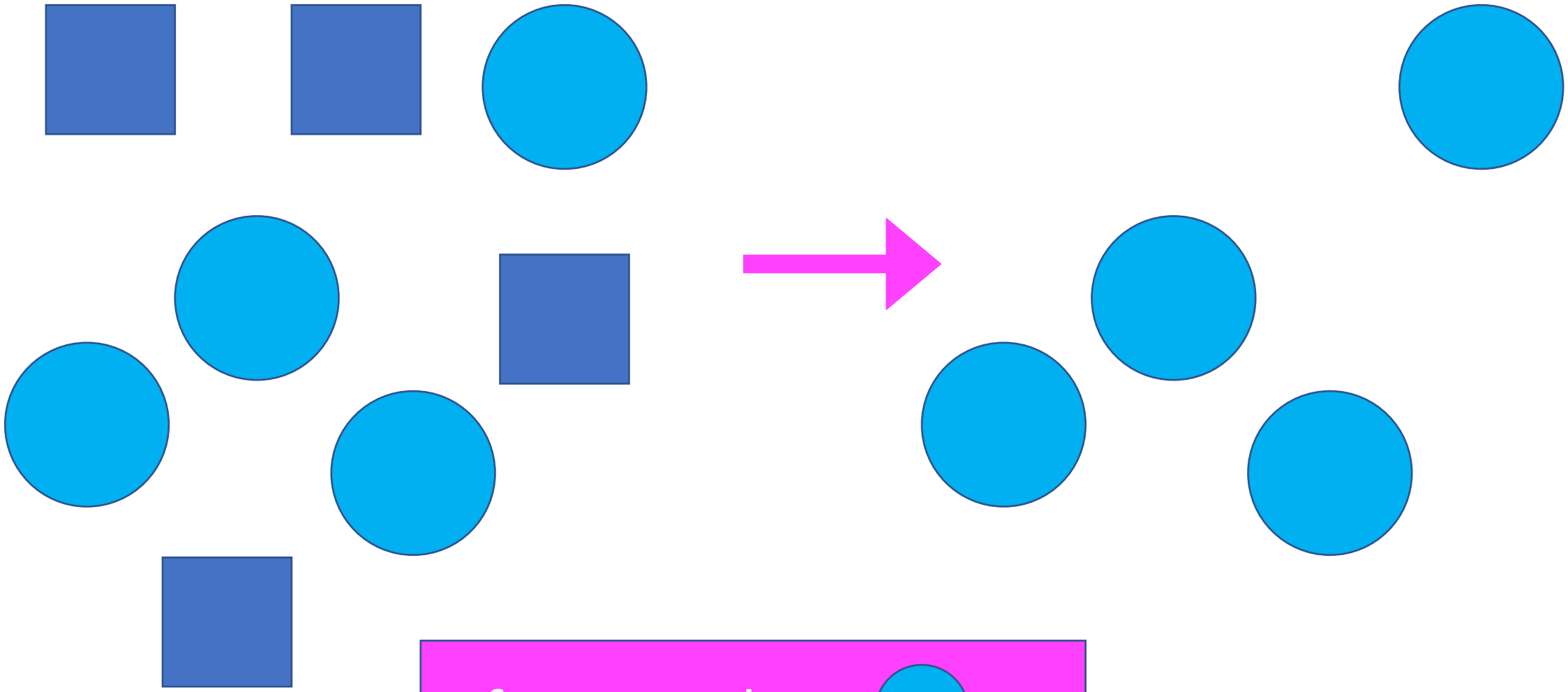


false consensus effect: reality

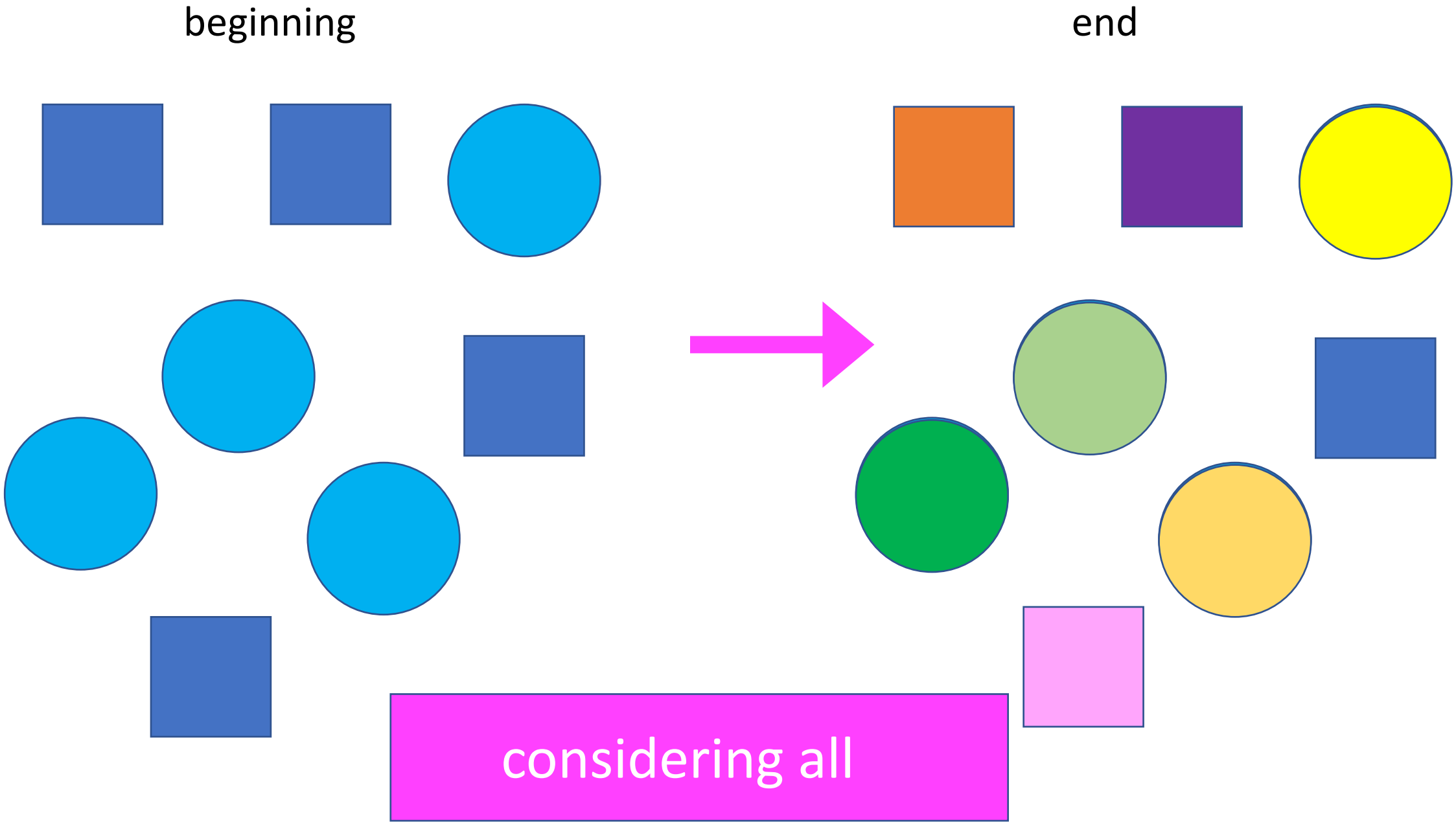


beginning

end



If you teach to s


beginning
end
considering all

Who are your students today?

Who do you
want your students to be at
the end of this course or program?

Who are your students now?

Who do future teachers
want your students to be at
the end of this course or program?

Who are your students now?

Who do future employers
want your students to be at
the end of this course or program?

Who are your students now?

Who do students
want to be at
the end of this course or program?

Who do we want MAT135 students to be after this course?

me: possess deep UNDERSTANDING of calculus

science profs: CONFIDENT to approach math in new contexts

students: be SUCCESSFUL

future employers: capable PROBLEM SOLVERS

community: DIVERSE, as a group

Begin with the end in mind.

- Stephen Covey

What do we want students to be able to do at the end of MAT135/136?

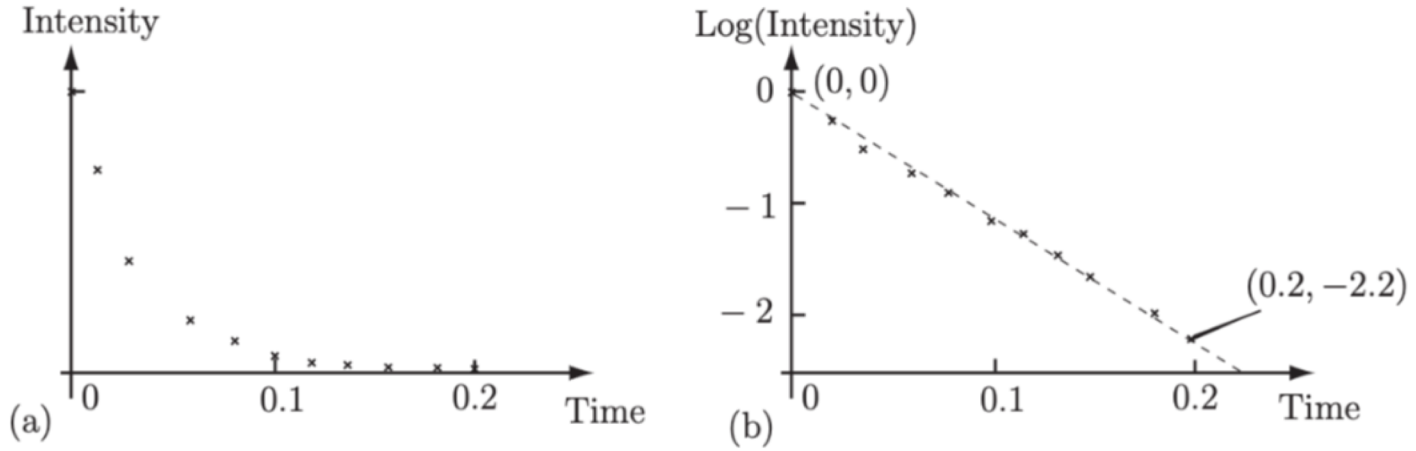
Identify & explain **overarching themes** of calculus

(5 points) Following a conference where experts discussed concepts that they believed were central to first-year calculus, Paul Zorn wrote: "The theme of approximation, it was agreed, is central to calculus – what it is and what it does."

Suppose that you are writing the introduction to a MAT136 tutorial guide that explains how the theme of approximation seen in MAT135 continues through MAT136. Briefly explain how the **algebraic (limit) definition** of the definite integral is related to **numerical approximations** of the integral. Support your answer with a **graphical representation** demonstrating different aspects of the definition.

Unpack **mathematics in context** (e.g. scientific papers).

(9 points) As part of a test of the acoustics of a concert hall, consultants asked a trombone player to play a single note at maximum volume. Once the sound had reached its maximum intensity, the player stopped and the sound intensity was measured for the next 0.2 seconds at regular intervals. The consultants scaled the intensity measurements so that the maximum intensity at time 0 was 1, and then plotted their data on graph (a) below.

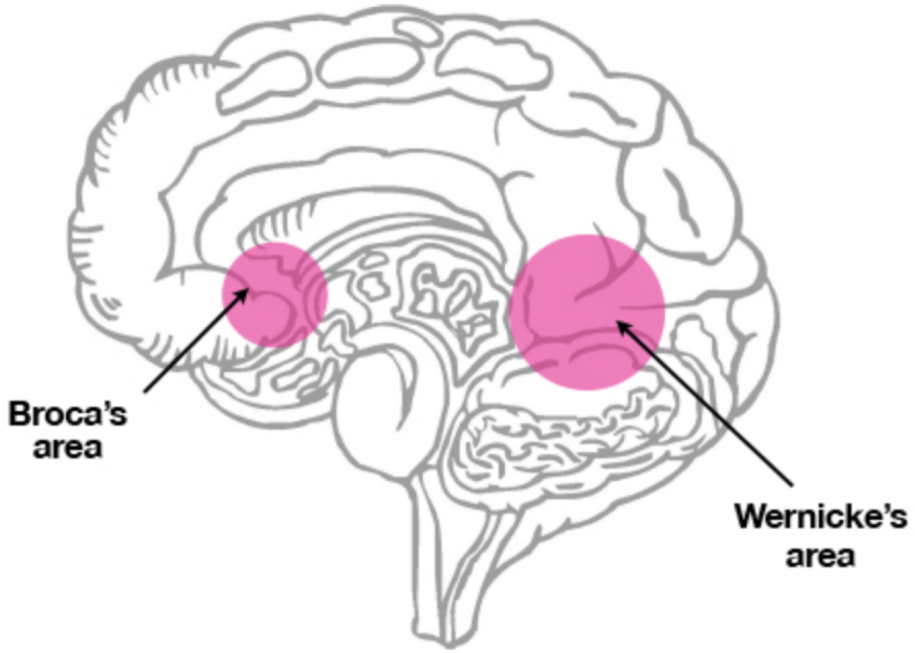


The consultants suspect that this data is best modelled by an exponential model of the form $I = I_0 10^{kt}$ where I is the relative intensity and t is time in seconds after the maximum intensity is reached. To check their suspicion, they plotted the graph (b) above, where the **logarithm is a base 10 logarithm**.

Design a mathematical story for your students

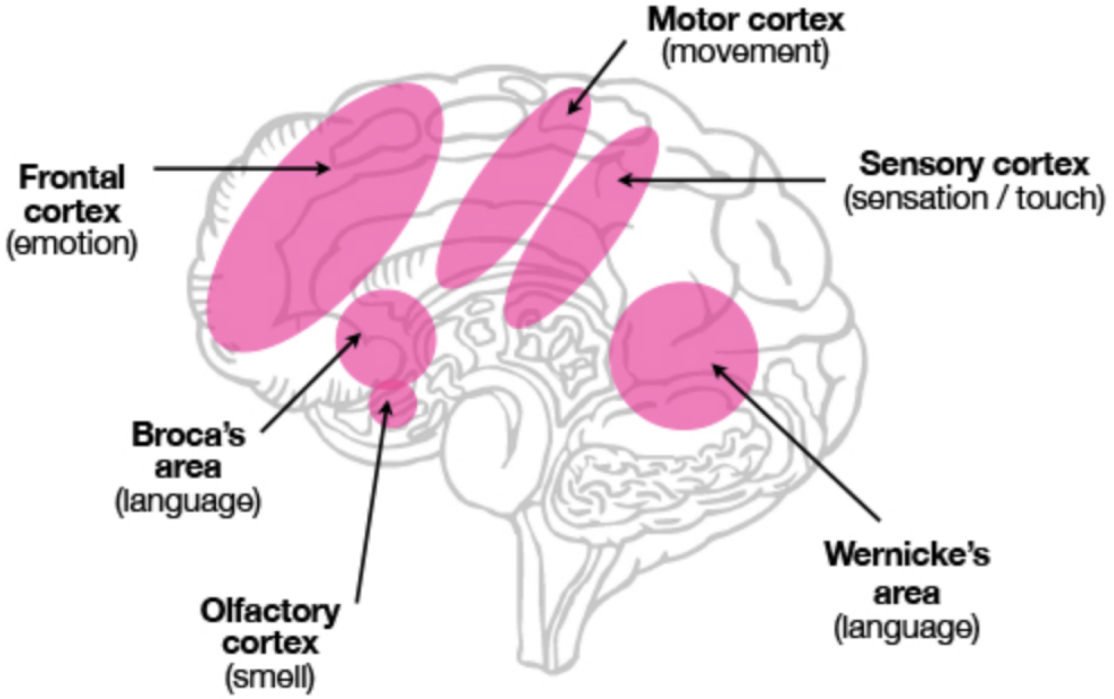
All humans like stories.

- Trish Hall



Brain exposed to facts

(making sense of language)



Brain exposed to stories

(all sense areas active)

Images from *What Science Says about the Effect of Stories on our Brains*, www.sarahdoody.com

For a student to learn what we intend to teach them, they must have a need for it, where "need" means intellectual need, not social or emotional need.

- Guershon Harel

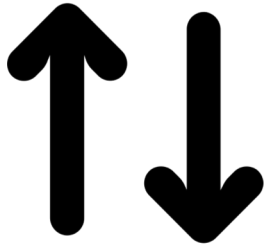
Harel, G. (2008b). DNR Perspective on Mathematics Curriculum and Instruction, Part II. *Zentralblatt fuer Didaktik der Mathematik* 40, 893-907.

Traditional Calculus Sequence

Differential Calculus	Limits Definition of the Derivative
Integral Calculus	Integrals Infinite Series Taylor Polynomials & Series

Traditional Calculus Sequence

Differential Calculus



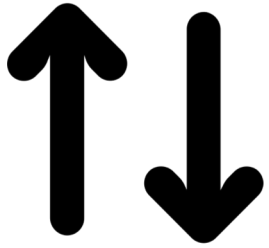
Definition of the Derivative
Limits

Integral Calculus

Integrals
Infinite Series
Taylor Polynomials & Series

Sequence in MAT135/136

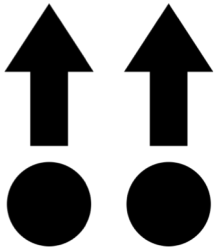
Differential Calculus



Definition of the Derivative

Limits

Integral Calculus



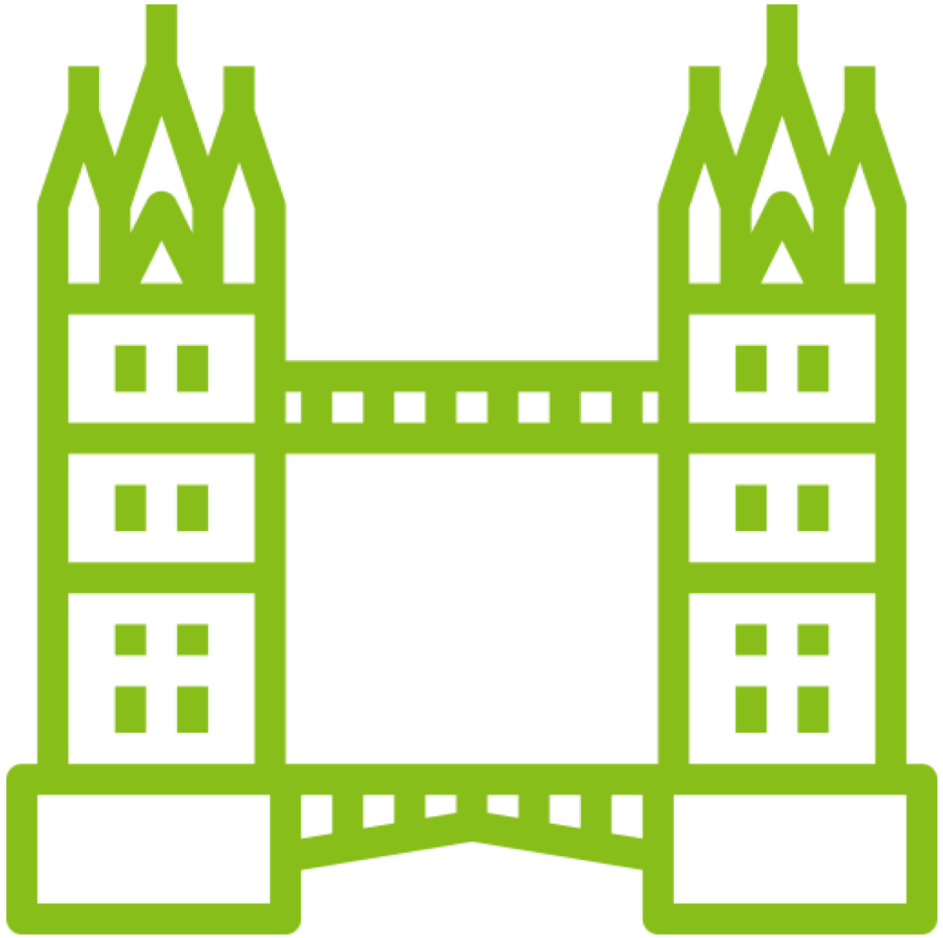
Motivation: Taylor Polynomials & Series

Integrals

Infinite Series

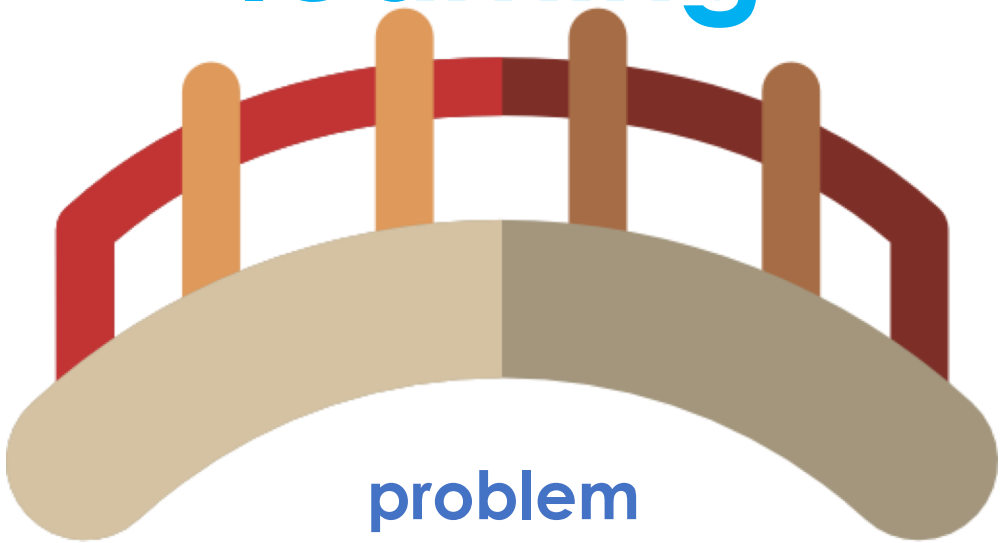
Exercise:

what comes before this topic? after?



Build a Bridge for students' learning

learning



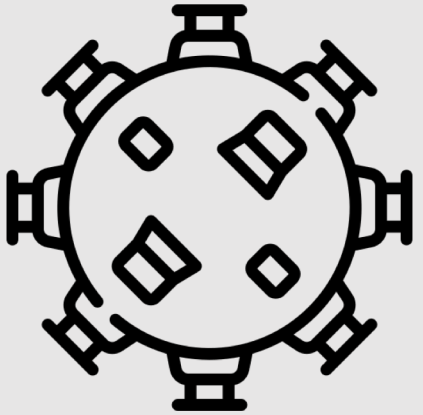
where students are

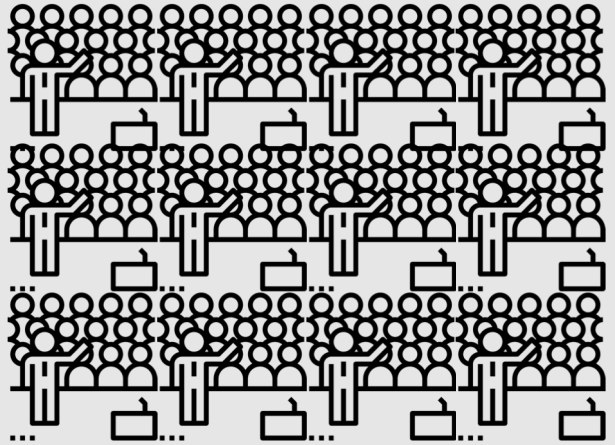
problem sets?

where we want students to be

the best bridge depends on context & students

Example: Problem Sets

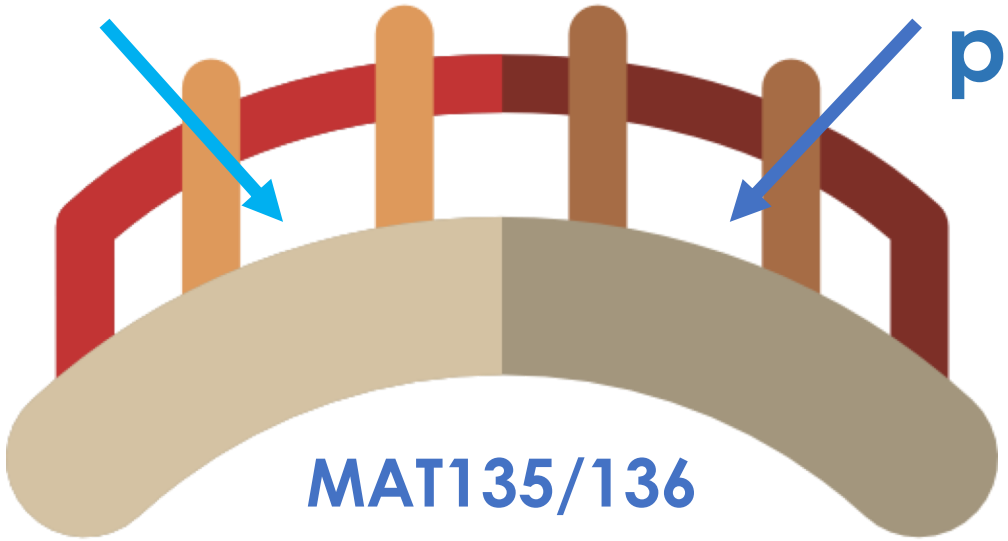




Substantive feedback		👎
Timely feedback		👎
Form they will use in the future?		
Future writing courses?		

Semester 1:
Build communication skills

Semester 2:
Scientific poster project



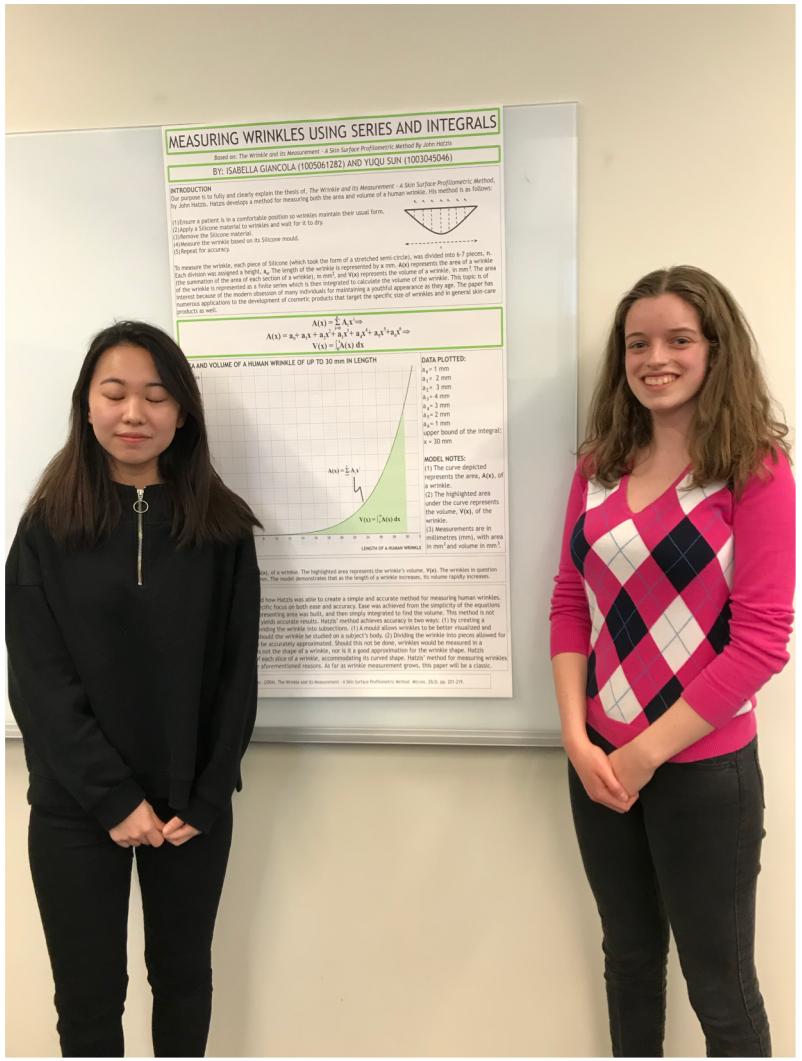
uncomfortable with math in unfamiliar contexts

strong understanding of math in novel scientific papers

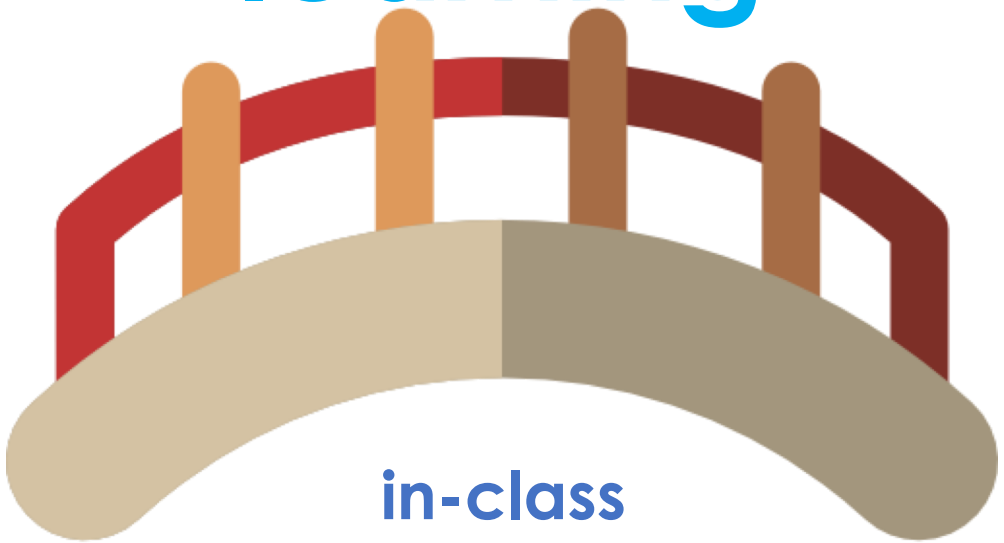
Example:

Scientific poster project in MAT136

- Based on scientific paper
- Describes context
- Highlights the use of an integral or derivative
- Includes a visual representation of that integral or derivative



learning



where students are

in-class activities?

where we want students to be

Classroom environments in which students are provided opportunities to engage in mathematical investigation, communication, and group problem-solving, while also receiving feedback on their work from both experts and peers, have a positive effect on learning… we call on institutions of higher education, mathematics departments and the mathematics faculty… to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms.

- Conference Board of the Mathematical Sciences Statement on Active Learning

2 simple active learning activities

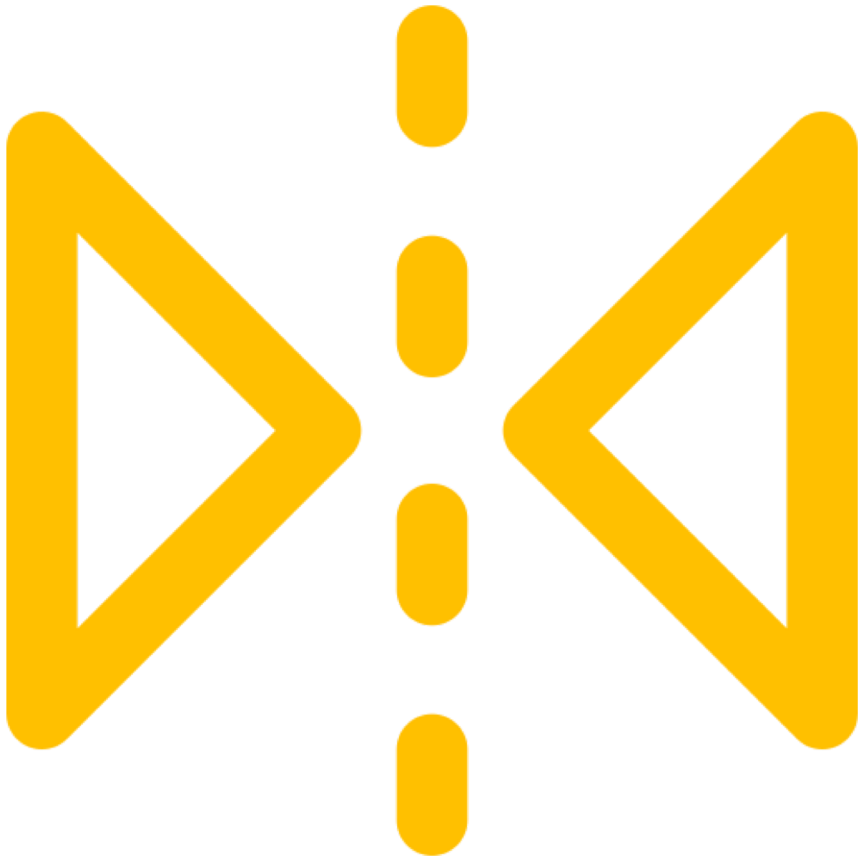
vote on answer to problem

think-pair-share

Active Learning techniques can play a particularly important role with regard to equity, diversity, and access in mathematics education.

Conference Board of the Mathematical Sciences

Statement on Active Learning



Advance & reflect

We can only understand through listening and reflection.

- Stephen Brookfield

Example: 1-minute paper prompts (CAT)

Name 1 thing about _____ you understand well and 1 thing that you are working to understand

When were you most engaged in class today? Most distanced?

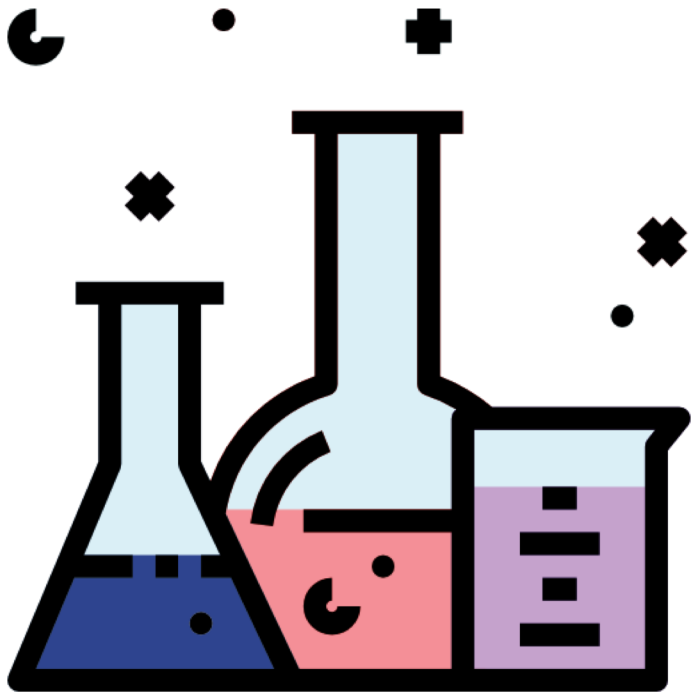
What are the 3 most important concepts from class today?

Example: Post-Class Reflection Questions

- What happened?
- Who made the class special?
- What surprised me the most?
- Did it live up to my expectations?

"People practicing critically reflective teaching ask questions and play around with different possibilities."

- Stephen Brookfield

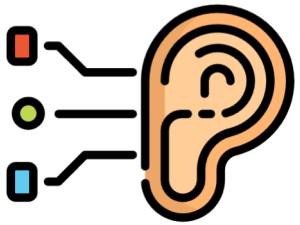


experiment!

seek help from experts

Example:

Confidence in Introductory Calculus



"I was good at math in high school, but failed the midterm. I guess I'm not cut out for university-level math."

How to prevent post-midterm drops?
How to increase confidence in large class?

MAT135-Academic Success Centre collaboration

caution

All practices are rooted in **instructors** listening to *their* students

caution

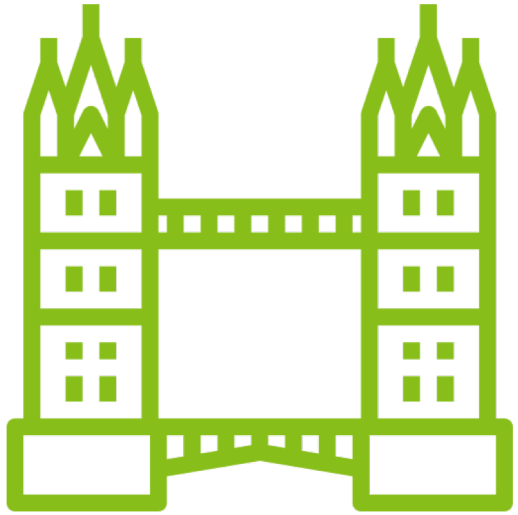
All practices are rooted in **instructors** listening to *their* students

Relying on students to assess instructors decreases faculty diversity.

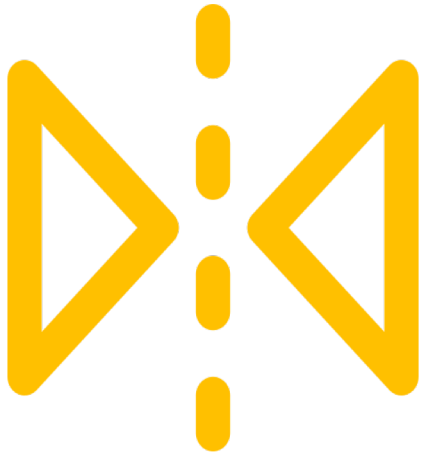
4 Ways to Build Mathematical Understanding & Diversity

Develop a vision of & for your students

Design a story for your students



Build bridge for students' learning



Advance & reflect

Courage
is the most important of
all the virtues.
Without courage you
can't practice any
other virtue consistently.

- Maya Angelou