

# Kasra Rafi

## Curriculum Vitae

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### Academic Positions

- 2018–present **Professor**, *University of Toronto*
- 2012–present **Member of School of Graduate Studies**, *University of Toronto*
- 2019–2021 **Associate Chair (Graduate)**, *University of Toronto*
- 2015–2018 **Associate Professor**, *University of Toronto*
- 2012–2015 **Assistant Professor**, *University of Toronto*
- 2008–2012 **Assistant Professor**, *University of Oklahoma*
- 2007–2008 **L.E. Dickson Instructor**, *University of Chicago*
- 2004–2007 **Postdoctoral Fellow**, *University of Connecticut*
- 2001–2004 **Ky Fan Visiting Assistant Professor**, *University of California Santa Barbara*

### Visiting Positions

- Fall 2022 Visiting Professor, *University of Chicago*
- Fall 2016 Simons Visiting Professor, *Mathematical Sciences Research Institute*
- May–Aug 2010 Hausdorff Research Institute for Mathematics
- Jan–Jul 2009 Clay Research Scholar, *University of Chicago*
- Fall 2007 *Mathematical Sciences Research Institute*
- Jul–Aug 2003 Isaac Newton Institute for Mathematical Sciences, *University of Cambridge, UK*

### Education

- 2001 **Ph.D., Mathematics**, *Stony Brook University*, Stony Brook, NY
- 1995 **B.Sc., Mathematics**, *Sharif Institute of Technology*, Tehran, Iran

### Grants and Recognitions

- 2024–2029 NSERC Discovery grant, RGPIN-05507
- 2024 Fellow of American Mathematical Society
- 2021–2022 Simons Fellow in Mathematics
- 2020 Fields Institute Fellow
- 2018–2024 NSERC Discovery grant, RGPIN 06486
- Fall 2016 Simons Visiting Professor at MSRI
- 2013–2018 NSERC Discovery grant, RGPIN 435885
- 2010–2013 NSF Research Grant PI, DMS-1007811
- Jan–Aug 2009 Clay Research Scholar, *University of Chicago*
- 2001 James Simons Prize for Excellence in Research, *Stony Brook University*
- 1992 Silver Medal, *International Mathematics Olympiad, Moscow*

## Fields of Interest

Geometry and dynamical properties of Teichmüller space  
Geometric group theory, in particular, mapping class groups and  $\text{Out}(\mathbb{F}_n)$   
Low dimensional Geometry and Topology

## Research papers

### Preprints

- 2024 **The quasi-redirecting boundary,**  
*with Yulan Qing*
- 2023 **Random walks on groups and superlinear divergent geodesics,**  
*Kunal Chawla, Inhyeok Choi and Vivian He*
- 2021 **Asymptotic Dimension of Big Mapping Class Groups,**  
*with Curtis Grant and Yvon Verberne*

### To Appear

- 2022 **Genericity of sublinearly Morse directions in CAT(0) spaces and the Teichmüller space,**  
*with Ilya Gekhtman and Yulan Qing,*  
to appear in *Mathematische Zeitschrift*
- 2022 **Fibered 3-manifolds and Veech groups,**  
*with Christopher J. Leininger, Nicholas Rouse, Emily Shinkle and Yvon Verberne,*  
to appear in *Algebraic & Geometric Topology*
- 2022 **Two results on end spaces of infinite type surfaces,**  
*with Kathryn Mann, Kasra Rafi,*  
to appear in *Michigan Mathematical Journal*

### Published

- 2024 **Sub-linearly Morse Boundary II: Proper Geodesic Spaces,**  
*with Yulan Qing and Giulio Tiozzo,*  
*Geometry & Topology* **28:4** (2024) 1829–1889
- 2023 **Large Scale Geometry of Big Mapping Class Groups,**  
*with Kathryn Mann,*  
*Geometry & Topology* **27:6** (2023), 2237–2296
- 2022 **Big mapping class groups with hyperbolic actions: classification and applications,**  
*with Camille Horbez and Yulan Qing,*  
*Journal of the Institute of Mathematics of Jussieu* **21** (2022), no. 6, 2173–2204
- 2022 **Benjamini-Schramm convergence of periodic orbits,**  
*with Amir Mohammadi,*  
*Geometriae Dedicata* **216** (2022), no. 5, Paper No. 57
- 2022 **Sub-linearly Morse Boundary I: CAT(0) Spaces,**  
*with Yulan Qing and Giulio Tiozzo,*  
*Advances in Mathematics* **404** (2022), part B, Paper No. 108442
- 2022 **Expected covering radius of a translation surface,**  
*with Howard Masur and Anja Randecker,*  
*International Mathematics Research Notices* 2022, no. 10, 7967–8002
- 2022 **Local maxima of the systole function,**  
*with Maxime Fortier Bourque,*  
*Journal of the European Mathematical Society (JEMS)* **24** (2022), no. 2, 623–668

- 2021 **Geodesics in the Mapping Class Group**,  
with Yvon Verberne,  
Algebraic & Geometric Topology **21** (2021), no. 6, 2995–3017
- 2021 **Convexity of balls in the outer space**,  
with Yulan Qing,  
Groups, Geometry, and Dynamics **15** (2021), no. 3, 893–934
- 2021 **Quasi-geodesics in  $\text{Out}(\mathbb{F}_n)$  and their shadows in sub-factors**,  
with Yulan Qing,  
Journal of the London Mathematical Society (2) **104** (2021), no. 1, 147–168
- 2020 **Centralizers in Mapping Class Group and decidability of Thurston Equivalence**,  
with Nikita Selinger and Michael Yampolsky,  
Arnold Mathematical Journal **6** (2020), no. 2, 271–290
- 2020 **Renormalizing an infinite rational IET**,  
with W. Patrick Hooper and Anja Randecker,  
Discrete and Continuous Dynamical Systems - Series A (2020), **40**, no. 9, 5105–5116
- 2020 **Coarse and fine geometry of the Thurston metric**,  
with David Dumas and Anna Lenzhen and Jing Tao,  
Forum of Mathematics, Sigma **8** (2020), paper no. e28, 58 pages
- 2020 **Isomorphisms between big mapping class groups**,  
with Juliette Bavard and Spencer Dowdall,  
International Mathematics Research Notices (2020), no. 10, 3084–3099
- 2020 **Limit sets of Weil-Petersson geodesics with non-minimal ending laminations**,  
with Jeffrey Brock, Chris Leininger and Babak Modami,  
Journal of Topology and Analysis **12** (2020), no. 1, 1–28
- 2020 **Limit sets of Teichmüller geodesics with minimal non-uniquely ergodic vertical foliation II**,  
with Jeffrey Brock, Chris Leininger and Babak Modami,  
Journal für reine und angewandte Mathematik, **758** (2020), 1–66
- 2019 **Limit sets of Weil-Petersson geodesics**,  
with Jeffrey Brock, Chris Leininger and Babak Modami,  
International Mathematics Research Notices (2019), no. 24, 7604–7658
- 2019 **Geodesic Currents and Counting Problems**,  
with Juan Souto,  
Geometric and Functional Analysis **29** (2019), no. 3, 871–889
- 2019 **Counting closed geodesics in strata**,  
with Alex Eskin and Maryam Mirzakhani,  
Inventiones Mathematicae **215** (2019), no. 2, 535–607
- 2018 **Teichmüller geodesics with  $d$ -dimensional limit sets**,  
with Anna Lenzhen and Babak Modami,  
Journal of Modern Dynamics **12** (2018), 261–283
- 2018 **Rigidity of Teichmüller space**,  
with Alex Eskin and Howard Masur,  
Geometry & Topology **22** (2018), no. 7, 4259–4306
- 2018 **Non-convex balls in the Teichmüller metric**,  
with Maxime Fortier Bourque,  
Journal of Differential Geometry **110** (2018), no. 3, 379–412
- 2018 **Algebraic ending laminations and quasi-convexity**,  
with Mahan Mj,  
Algebraic & Geometric Topology **18** (2018), no. 4, 1883–1916

- 2018 **Limit sets of Teichmüller geodesics with minimal non-uniquely ergodic vertical foliation**,  
with *Chris Leininger* and *Anna Lenzhen*,  
*Journal für die reine und angewandte Mathematik* **737** (2018), 1–32
- 2017 **Uniform fellow traveling between surgery paths in the sphere graph**,  
with *Matt Clay* and *Yulan Qing*,  
*Algebraic & Geometric Topology* **17**, (2017), no. 6, 3751–3778
- 2017 **Large scale rank of Teichmüller space**,  
with *Alex Eskin* and *Howard Masur*,  
*Duke Mathematical Journal* **166** (2017), no. 8, 1517–1572
- 2016 **Uniform growth rate**,  
with *Jing Tao*,  
*Proceedings of the American Mathematical Society* **144** (2016), no. 4, 1415–1427
- 2015 **The shadow of a Thurston geodesic to the curve graph**,  
with *Anna Lenzhen* and *Jing Tao*,  
*Journal of Topology* **8** (2015), no. 4, 1085–1118
- 2015 **Digraphs and cycle polynomials for free-by-cyclic groups**,  
with *Yael Algom-Kfir* and *Eriko Hironaka*,  
*Geometry & Topology* **19** (2015), no 2, 1111–1154
- 2015 **Mapping tori of small dilatation expanding train-track maps**,  
with *Yael Algom Kfir*,  
*Topology and its Applications* **180** (2015), 44–63
- 2014 **Uniform hyperbolicity of the curve graph via surgery sequences**,  
with *Matt Clay* and *Saul Schleimer*,  
*Algebraic & Geometric Topology* **14** (2014), no. 6, 3325–3344
- 2014 **Hyperbolicity in Teichmüller space**,  
*Geometry & Topology* **18** (2014), no 5, 3025–3053
- 2014 **On hyperbolicity of free splitting and free factor complexes**,  
with *Ilya Kapovich*,  
*Groups, Geometry, and Dynamics* **8** (2014), no. 2, 391–414
- 2013 **Diameter of the thick part of moduli space**,  
with *Jing Tao*,  
*Duke Mathematical Journal* **162** (2013), no. 10, 1833–1876
- 2012 **Bounded combinatorics and the Lipschitz metric on Teichmüller space**,  
with *Anna Lenzhen* and *Jing Tao*,  
*Geometriae Dedicata* **159** (2012), 353–371
- 2012 **Grafting rays fellow travel Teichmüller geodesics**,  
with *Young-Eun Choi* and *David Dumas*,  
*International Mathematics Research Notices* (2012), no. 11, 2445–2492
- 2011 **Length of a curve is quasi-convex along a Teichmüller geodesic**,  
with *Anna Lenzhen*,  
*Journal of Differential Geometry* **88** (2011), no. 2, 267–295
- 2011 **Curve complexes are rigid**,  
with *Saul Schleimer*,  
*Duke Mathematical Journal* Volume **158** (2011), no 2, 225–246
- 2010 **Length spectra and degeneration of flat metrics**,  
with *Moon Duchin* and *Christopher Leininger*,  
*Inventiones Mathematicae*, **182**, (2010) no. 2, 231–277
- 2009 **Divergence rate of geodesics in Teichmüller space and mapping class groups**,  
with *Moon Duchin*,  
*Geometric and Functional Analysis* **19** (2009), no. 3, 722–742

- 2009 **Covers and the curve complex**,  
with *Saul Schleimer*,  
*Geometry & Topology* **13** (2009), no. 4, 2141–2162
- 2008 **Lines of minima and Teichmüller geodesics**,  
with *Young-Eun Choi and Caroline Series*,  
*Geometric and Functional Analysis* **18** (2008), no. 3, 698–754
- 2008 **Lines of minima are uniformly quasigeodesic**,  
with *Young-Eun Choi and Caroline Series*,  
*Pacific Journal of Mathematics* **237** (2008), no. 1, 21–44
- 2007 **Comparison between Teichmüller and Lipschitz metrics**,  
with *Young-Eun Choi*,  
*Journal of the London Mathematical Society* (2) **76** (2007), no. 3, 739–756
- 2007 **A combinatorial model for the Teichmüller metric**,  
*Geometric and Functional Analysis* **17** (2007), no. 3, 936–959
- 2007 **Thick-thin decomposition for quadratic differentials**,  
*Mathematical Research Letters* **14** (2007), no. 2, 333–341
- 2005 **A characterization of short curves of a Teichmüller geodesic**,  
*Geometry & Topology* **9** (2005), 179–202

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## Invited Talks

### Lecture Series

- June 2023 **Mirzakhani's integration formula for the volume of moduli space**, *Geometry and spectra of random hyperbolic surfaces, Université de Montréal*
- May 2023 **Combinatorial Teichmüller Theory**, *Beijing International Center for Mathematical Research*
- March 2022 **Course Geometry of Big Mapping Class Groups**, *Shanghai Center for Mathematical Sciences*
- March 2022 **Sublinearly Morse Boundary**, *A Week at Infinity, Online workshop*

### Recent Conferences

- June 2024 **Inter-University Centre Dubrovnik**, *Dubrovnik, Croatia*, Dubrovnik X - Topology & Dynamical Systems
- July 2023 **Erwin Schrödinger International Institute for Mathematics**, *Vienna, Austria*, Geometric and Asymptotic Group Theory with Applications 2023 - Groups and Dynamics
- January 2023 **University of Utah, Slat Lake City**, *Salt Flat: A research community in Teichmüller dynamics*
- December 2022 **PIMS**, *Vancouver*, The 2022 Pacific Rim Mathematical Association Congress
- Sept. 2022 **CIRM**, *Marseille Luminy, France*, Dynamics and Geometry on Moduli Spaces conference
- August 2022 **Park City**, *Utah*, Wasatch Topology Conference
- April 2022 **AIM**, *San Jose*, Random walks beyond hyperbolic groups
- January 2020 **Tsinghua Sanya International Mathematics Forum**, *Sanya, China*, Frontier of Analysis and Geometry of Teichmüller Spaces
- June 2019 **Bowdoin College**, *Brunswick*, Boundaries of Random Walks and Applications
- August 2018 **Fields Institute**, *Toronto*, Workshop on Geometry of Teichmüller Space
- July 2018 **Peking University**, *Beijing*, A conference on Geometric Group Theory
- June 2018 **Fields Institute**, *Toronto*, 2018 Fields Institute Annual General Meeting
- May 2018 **Stanford University**, *Palo Alto*, The Mathematical Legacy of Maryam Mirzakhani
- April 2018 **University of Warwick**, *Coventry*, Geometry of Teichmüller space and mapping class groups
- Nov. 2017 **ICTS**, *Bangalore*, Surface group representation and geometric structures
- Sept. 2017 **Ventotene**, *Italy*, Moduli Space

- August 2017 **Trinity College, Dublin**, Geometry and Dynamics of Moduli Spaces
- June 2017 **Newton Institute, Cambridge**, Group actions and cohomology in non-positive curvature
- May 2017 **Vanderbilt University, Vanderbilt**, Shank Conference
- April 2017 **University of Arkansas, Fayetteville**, Redbud Topology Conference
- Feb. 2017 **CIRM, Marseille**, Dynamics and Geometry in the Teichmüller Space
- October 2016 **MSRI, Berkeley**, Geometry of mapping class groups and  $\text{Out}(\mathbb{F}_n)$
- July 2016 **KAIST, Daejeon, Korea**, Geometric Topology Fair
- May 2016 **ICERM, Providence**, Effective and Algorithmic Methods in Hyperbolic Geometry and Free Groups
- May 2016 **Casa Matemática Oaxaca, Oaxaca, Mexico**, Flat Surfaces and Dynamics of Moduli Space
- Jan. 2016 **African Institute for Mathematical Sciences, Cape Town**, Quadratic Differentials
- Sept. 2015 **Mathematical Institute, Oxford, UK**, Geometry and Dynamics on Moduli Spaces
- June 2015 **Inter-University Centre Dubrovnik, Dubrovnik, Croatia**, Dubrovnik VIII - Geometric Topology, Geometric Group Theory & Dynamical Systems
- May 2015 **University of Texas at Austin, Austin**, Workshop on Mapping Class Groups and  $\text{Out}(\mathbb{F}_n)$

### Online Seminars

- April 2021 **Frontiers in Mathematical Sciences, IPM, Iran**
- Sept. 2020 **Big Surfaces Seminar**
- July 2020 **Australian Geometric Topology Webinar**
- April 2020 **Billiards and Surfaces á la Teichmüller and Riemann**

### Colloquia (Excluding job talks)

- 2023 **Tel Aviv University**
- 2020 **Rice University**
- 2019 **University of Toronto**
- 2017 **Pittsburgh University**
- 2017 **University of Michigan**
- 2014 **University of Oklahoma**
- 2014 **University of Indiana**
- 2011 **University of Arkansas**
- 2011 **Rice University**

### Seminars

Brown University, California Institute of Technology, Cornell University, CUNY Graduate Center, Columbia University, Harvard University, Institut des Hautes Études Scientifiques, Joint USC-UCLA-Caltech, Kyoto University, McGill University, Osaka University, Oklahoma State University, Princeton University, Rutgers University, South China University of Technology, Stanford University, Sun Yat-Sen University, SUNY at Stony Brook, Tufts University, Université de Rennes, Université Paris-Sud, University of Connecticut, University of Maryland, University of California at Davis, University of California at Santa Barbara, University of Chicago, University of Illinois at Chicago, University of Illinois at Urbana-Champaign, University of Michigan, University of Southern California, University of Seoul, University of Texas Austin, University of Toronto, University of Utah, University of Warwick, University of Wisconsin, Yale University.

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## Supervision

### Graduate students

Current Richard Cao  
Current Amaranta Martinez De La Rosa  
Current Vivian He  
Current Ali Pirhosseinloo  
Current Mohammad Honari  
2023 Pooya Honaryar  
2022 Assaf Bar-Natan, University of Toronto  
2021 Ivan Telpukhovskiy, University of Toronto  
2020 Yvon Verberne, University of Toronto  
2013 Funda Gültepe, University of Oklahoma  
2013 Yoe Alexander Herrera Jaramillo, University of Oklahoma

### Undergraduate Research

2023 Michael Li, (Supported by NSERC)  
2023 Diba Heydari, (Supported by NSERC)  
2023 Summer Sun, (Independent Research)  
2022 Asha McMullin, (Independent Research)  
2022 Kunal Chawla, (Supported by NSERC)  
2021 Vivian He, (Supported by NSERC)  
2019 Curtis Grant, (Supported by NSERC)  
2018 Ci Zhang, (Supported by UT Work Study Program)  
2018 Jordan Hofmann, (Supported by NSERC)  
2017 Adriano Pacifico, (Supported by NSERC)

### Highschool Mentorship

2019 Anna Krokline  
2018 Clare Radnoff  
2016 Gemma Zhang  
2015 Abtin Ameri  
2015 Andrey Khesin  
2014 Coco Zhang

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## Teaching Experience

### Undergraduate courses

Pre-calculus, Single and Multivariable Calculus at all levels, Honors Calculus, Ordinary and Partial Differential Equations, Physical Mathematics, Discrete Mathematics, Aha! Mathematical Discovery and Creative Problem Solving, Logic and Proofs, Linear Algebra, Abstract Algebra, Probability, Graph theory, Complex Analysis, General Topology, Knot Theory, Geometric Group theory, Problem Solving Seminar, and Seminar in Mathematics.

### Graduate courses

Algebra for Teachers, Geometry for Teachers, Introduction to Hyperbolic Geometry, Algebraic Topology, Differential Topology, Combinatorial Methods and Geometric Literacy.

### Topics courses

Kleinian groups, Introduction to Teichmüller Space, Currents and Counting problems, Geometry and Dynamics in Teichmüller Space Equipped with the Thurston Metric,  $SL(2, \mathbb{R})$ , combinatorial Teichmüller theory.

## Recent Professional Activities

- May 2023 Co-organizer for the conference: *1,2,3: Curves, Surfaces, and 3-Manifolds*, CMS, Haifa, Israel.
- January 2023 Co-organizer for the conference: *Salt Flat: A research community in Teichmüller dynamics*, University of Utah, Slat Lake City.
- April 2022 Co-organizer for the conference *Mapping class groups and  $Out(F_n)$* , Institut Henri Poincaré, Paris.
- May 2019 Co-organizer of the workshop *Flat Surfaces and Dynamics of Moduli Space, II*, Oaxaca.
- Fall 2018 Co-organizer of the Thematic Program on *Teichmüller Theory and its Connections to Geometry, Topology and Dynamics* Fields Institute, Toronto.
- Nov 2018 Co-organizer for the *Fields Symposium in honor of Maryam Mirzakhani*, Fields Institute, Toronto.
- 2024-present Associate Editor for *Geometriae Dedicata*
- 2018-present Associate Editor for *The Journal of Iranian Mathematical Society*
- 2016-2019 Associate Editor for *The Journal of the Ramanujan Mathematical Society*
- 2008-2018 Mentor for Association for Women in Mathematics (AWM) Mentor Network