

# Kasra Rafi

## Curriculum Vitae

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### Academic Positions

- 2019–present **Associate Chair (Graduate)**, *University of Toronto.*
- 2018–present **Professor**, *University of Toronto.*
- 2015–2018 **Associate Professor**, *University of Toronto.*
- 2012–2015 **Assistant Professor**, *University of Toronto.*
- 2012–present **Member of School of Graduate Studies**, *University of Toronto.*
- 2008–2012 **Assistant Professor**, *University of Oklahoma.*
- 2007–2008 **L.E. Dickson Instructor**, *University of Chicago.*
- 2004–2007 **Postdoctoral Fellow**, *University of Connecticut.*
- 2001–2004 **Ky Fan Visiting Assistant Professor**, *University of California Santa Barbara.*

### Visiting Positions

- Fall 2016 Simons Visiting Professor at Mathematical Sciences Research Institute
- May-Aug 2010 Hausdorff Research Institute for Mathematics
- Jan-Jul 2009 Clay Research Scholar, University of Chicago
- Fall 2007 Mathematical Sciences Research Institute
- Jul-Aug 2003 Isaac Newton Institute for Mathematical Sciences, University of Cambridge, UK

### Education

- 2001 **Ph.D., Mathematics**, *Stony Brook University*, Stony Brook, NY.
- 1995 **B.Sc., Mathematics**, *Sharif Institute of Technology*, Tehran, Iran.

### Grants and Recognitions

- Fall 2016 Simons Visiting Professor at MSRI
- 2018–2023 NSERC Discovery grant, RGPIN 06486
- 2013–2018 NSERC Discovery grant, RGPIN 435885
- 2010–2013 NSF Research Grant PI, DMS-1007811
- Jan-Aug 2009 Clay Research Scholar, University of Chicago
- 2001 James Simons Prize for Excellence in Research, Stony Brook University
- 1992 Silver Medal, International Mathematics Olympiad, Moscow

### Fields of Interest

- Geometry and dynamical properties of Teichmüller space
- Geometric group theory, in particular, mapping class group and  $\text{Out}(\mathbb{F}_n)$
- Low dimensional Geometry and Topology

## Research papers

### Preprints

- 2019 **Sub-linearly Morse Boundary I: CAT(0) Spaces**, with *Yulan Qing and Giulio Tiozzo*.
- 2019 **Centralizers in Mapping Class Group and decidability of Thurston Equivalence**, with *Nikita Selinger and Michael Yampolsky*.
- 2018 **The shape of a generic translation surface in the minimal stratum**, with *Howard Masur and Anja Randecker*.
- 2018 **Renormalizing an infinite rational IET**, with *W. Patrick Hooper and Anja Randecker*.
- 2018 **Local maxima of the systole function**, with *Maxime Fortier Bourque*.
- 2018 **Quasi-geodesics in  $\text{Out}(\mathbb{F}_n)$  and their shadows in sub-factors**, with *Yulan Qing*.
- 2017 **Convexity of balls in the outer space**, with *Yulan Qing*.
- 2016 **Coarse and fine geometry of the Thurston metric**, with *David Dumas and Anna Lenzhen and Jing Tao*.

### To Appear

- 2017 **Limit sets of Weil-Petersson geodesics with non-minimal ending laminations**, with *Jeffrey Brock, Chris Leininger and Babak Modami*, to appear in *Journal of Topology and Analysis*.
- 2017 **Isomorphisms between big mapping class groups**, with *Juliette Bavard and Spencer Dowdall*, to appear in *International Mathematics Research Notices*.
- 2016 **Non-convex balls in the Teichmüller metric**, with *Maxime Fortier Bourque*, to appear in *Journal of Differential Geometry*.
- 2016 **Limit sets of Teichmüller geodesics with minimal non-uniquely ergodic vertical foliation II**, with *Jeffrey Brock, Chris Leininger and Babak Modami*, to appear in *Crelle's Journal*.

### Published

- 2019 **Geodesic Currents and Counting Problems**, with *Juan Souto*, *Geom. Funct. Anal.* **29** (2019), no. 3, 871–889.
- 2019 **Counting closed geodesics in strata**, with *Alex Eskin and Maryam Mirzakhani*, *Invent. Math.* **215** (2019), no. 2, 535–607.
- 2018 **Limit sets of Weil-Petersson geodesics**, with *Jeffrey Brock, Chris Leininger and Babak Modami*, *International Mathematics Research Notices*, published online <https://doi.org/10.1093/imrn/rny002>.
- 2018 **Teichmüller geodesics with  $d$ -dimensional limit sets**, with *Anna Lenzhen and Babak Modami*, *J. Mod. Dyn.* **12** (2018), 261–283.
- 2018 **Rigidity of Teichmüller space**, with *Alex Eskin and Howard Masur*, *Geom. Topol.* **22** (2018), no. 7, 4259–4306.
- 2018 **Non-convex balls in the Teichmüller metric**, with *Maxime Fortier Bourque*, *J. Differential Geom.* **110** (2018), no. 3, 379–412.
- 2018 **Algebraic ending laminations and quasi-convexity**, with *Mahan Mj*, *Algebraic & Geometric Topology* **18** (2018), no. 4, 1883–1916.
- 2018 **Limit sets of Teichmüller geodesics with minimal non-uniquely ergodic vertical foliation**, with *Chris Leininger and Anna Lenzhen*, *J. Reine Angew. Math. (Crelle's Journal)* **737** (2018), 1–32.
- 2017 **Uniform fellow traveling between surgery paths in the sphere graph**, with *Matt Clay and Yulan Qing*, *Algebraic & Geometric Topology* **17**, (2017), no. 6, 3751–3778.

- 2017 **Large scale rank of Teichmüller space**, with *Alex Eskin and Howard Masur*, *Duke Math. J.* **166** (2017), no. 8, 1517–1572.
- 2016 **Uniform growth rate**, with *Jing Tao*, *Proc. Amer. Math. Soc.* **144** (2016), no. 4, 1415–1427.
- 2015 **The shadow of a Thurston geodesic to the curve graph**, with *Anna Lenzhen and Jing Tao*, *J. Topol.* **8** (2015), no. 4, 1085–1118.
- 2015 **Digraphs and cycle polynomials for free-by-cyclic groups**, with *Yael Algom-Kfir and Eriko Hironaka*, *Geometry & Topology* **19** (2015), no 2, 1111–1154.
- 2015 **Mapping tori of small dilatation expanding train-track maps**, with *Yael Algom Kfir*, *Topology and its Applications* **180** (2015), 44–63.
- 2014 **Uniform hyperbolicity of the curve graph via surgery sequences**, with *Matt Clay and Saul Schleimer*, *Algebraic & Geometric Topology* **14** (2014), no. 6, 3325–3344.
- 2014 **Hyperbolicity in Teichmüller space**, *Geometry & Topology* **18** (2014), no 5, 3025–3053.
- 2014 **On hyperbolicity of free splitting and free factor complexes**, with *Ilya Kapovich*, *Groups Geom. Dyn.* **8** (2014), no. 2, 391–414.
- 2013 **Diameter of the thick part of moduli space**, with *Jing Tao*, *Duke Math. J.* **162** (2013), no. 10, 1833–1876.
- 2012 **Bounded combinatorics and the Lipschitz metric on Teichmüller space**, with *Anna Lenzhen and Jing Tao*, *Geom. Dedicata* **159** (2012), 353–371.
- 2012 **Grafting rays fellow travel Teichmüller geodesics**, with *Young-Eun Choi and David Dumas*, *Int. Math. Res. Not. IMRN* (2012), no. 11, 2445–2492.
- 2011 **Length of a curve is quasi-convex along a Teichmüller geodesic**, with *Anna Lenzhen*, *J. Differential Geom.* **88** (2011), no. 2, 267–295.
- 2011 **Curve complexes are rigid**, with *Saul Schleimer*, *Duke Math. J. Volume* **158** (2011), no 2, 225–246.
- 2010 **Length spectra and degeneration of flat metrics**, with *Moon Duchin and Christopher Leininger*, *Inventiones Mathematicae*, **182**, (2010) no. 2, 231–277.
- 2009 **Divergence rate of geodesics in Teichmüller space and mapping class groups**, with *Moon Duchin*, *Geom. Funct. Anal.* **19** (2009), no. 3, 722–742.
- 2009 **Covers and the curve complex**, with *Saul Schleimer*, *Geom. Topol.* **13** (2009), no. 4, 2141–2162.
- 2008 **Lines of minima and Teichmüller geodesics**, with *Young-Eun Choi and Caroline Series*, *Geom. Funct. Anal.* **18** (2008), no. 3, 698–754.
- 2008 **Lines of minima are uniformly quasigeodesic**, with *Young-Eun Choi and Caroline Series*, *Pacific J. Math.* **237** (2008), no. 1, 21–44.
- 2007 **Comparison between Teichmüller and Lipschitz metrics**, with *Young-Eun Choi*, *J. London Math. Soc. (2)* **76** (2007), no. 3, 739–756.
- 2007 **A combinatorial model for the Teichmüller metric**, *Geom. Funct. Anal.* **17** (2007), no. 3, 936–959.
- 2007 **Thick-thin decomposition for quadratic differentials**, *Math. Res. Lett.* **14** (2007), no. 2, 333–341.
- 2005 **A characterization of short curves of a Teichmüller geodesic**, *Geom. Topol.* **9** (2005), 179–202.

## Invited Talks

### Recent Conferences

- January 2020 **Tsinghua Sanya International Mathematics Forum**, *Sanya, China*, Frontier of Analysis and Geometry of Teichmüller Spaces.
- June 2019 **Bowdoin College**, *Brunswick*, Boundaries of Random Walks and Applications.
- August 2018 **Fields Institute**, *Toronto*, Workshop on Geometry of Teichmüller Space.
- July 2018 **Peking University**, *Beijing*, A conference on Geometric Group Theory.
- June 2018 **Fields Institute**, *Toronto*, 2018 Fields Institute Annual General Meeting.
- May 2018 **Stanford University**, *Palo Alto*, The Mathematical Legacy of Maryam Mirzakhani.
- April 2018 **University of Warwick**, *Coventry*, Geometry of Teichmüller space and mapping class groups.
- Nov. 2017 **ICTS**, *Bangalore*, Surface group representation and geometric structures.
- Sept. 2017 **Ventotene**, *Italy*, Moduli Space.
- August 2017 **Trinity College**, *Dublin*, Geometry and Dynamics of Moduli Spaces.
- June 2017 **Newton Institute**, *Cambridge*, Group actions and cohomology in non-positive curvature.
- May 2017 **Vanderbilt University**, *Vanderbilt*, Shank Conference.
- April 2017 **University of Arkansas**, *Fayetteville*, Redbud Topology Conference.
- Feb. 2017 **CIRM**, *Marseille*, Dynamics and Geometry in the Teichmüller Space.
- October 2016 **MSRI**, *Berkeley*, Geometry of mapping class groups and  $\text{Out}(\mathbb{F}_n)$ .
- July 2016 **KAIST**, *Daejeon, Korea*, Geometric Topology Fair.
- May 2016 **ICERM**, *Providence*, Effective and Algorithmic Methods in Hyperbolic Geometry and Free Groups.
- May 2016 **Casa Matemática Oaxaca**, *Oaxaca, Mexico*, Flat Surfaces and Dynamics of Moduli Space.
- Jan. 2016 **African Institute for Mathematical Sciences**, *Cape Town*, Quadratic Differentials.
- Sept. 2015 **Mathematical Institute**, *Oxford, UK*, Geometry and Dynamics on Moduli Spaces.
- June 2015 **Inter-University Centre Dubrovnik**, *Dubrovnik, Croatia*, Dubrovnik VIII - Geometric Topology, Geometric Group Theory & Dynamical Systems.
- May 2015 **University of Texas at Austin**, *Austin*, Workshop on Mapping Class Groups and  $\text{Out}(\mathbb{F}_n)$ .

### Colloquia (Excluding job talks)

- 2017 **Pittsburgh University**.
- 2017 **University of Michigan**.
- 2014 **University of Oklahoma**.
- 2014 **University of Indiana**.
- 2011 **University of Arkansas**.
- 2011 **Rice University**.

### Seminars

Brown University, California Institute of Technology, Cornell University, CUNY Graduate Center, Columbia University, Harvard University, Institut des Hautes Études Scientifiques, Joint USC-UCLA-Caltech, Kyoto University, McGill University, Osaka University, Oklahoma State University, Princeton University, Rutgers University, Stanford University, SUNY at Stony Brook, Tufts University, Université de Rennes, Université Paris-Sud, University of Connecticut, University of Maryland, University of California at Davis, University of California at Santa Barbara, University of Chicago, University of Illinois at Chicago, University of Illinois at Urbana-Champaign, University of Michigan, University of Southern California, University of Seoul, University of Texas Austin, University of Toronto, University of Utah, University of Warwick, Yale University.

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## Supervision

### Graduate students

- Current Assaf Bar-Natan
- Current Thomas Dykes
- Current Pooya Honaryar
- Current Ivan Telpukhovskiy
- Current Yvon Verberne
  - 2013 Funda Gültepe, University of Oklahoma
  - 2013 Yoe Alexander Herrera Jaramillo, University of Oklahoma

### Undergraduate

- 2019 Curtis Grant
- 2018 Ci Zhang
- 2018 Jordan Hofmann
- 2017 Adriano Pacifico

### Highschool Mentorship

- 2019 Anna Krokline
- 2018 Clare Radnoff
- 2016 Gemma Zhang
- 2015 Abtin Ameri
- 2015 Andrey Khesin
- 2014 Coco Zhang

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## Teaching Experience

### Undergraduate courses

Pre-calculus, Single and Multivariable Calculus at all levels, Honors Calculus, Ordinary and Partial Differential Equations, Physical Mathematics, Discrete Mathematics, Logic and Proofs, Linear Algebra, Abstract Algebra, Probability, Graph theory, Complex Analysis, General Topology, Knot Theory, Problem Solving Seminar, and Seminar in Mathematics.

### Graduate courses

Algebra for Teachers, Geometry for Teachers, Introduction to Hyperbolic Geometry, Algebraic Topology, Differential Topology, Combinatorial Methods and Geometric Literacy.

### Topics courses

Kleinian groups, Introduction to Teichmüller Space, Currents and Counting problems, Geometry and Dynamics in Teichmüller Space Equipped with the Thurston Metric

## Recent Professional Activities

- May 2019 Co-organizer of the workshop *Flat Surfaces and Dynamics of Moduli Space, II*, Oaxaca.
- Fall 2018 Co-organizer of the Thematic Program on *Teichmüller Theory and its Connections to Geometry, Topology and Dynamics* Fields Institute, Toronto.
- Nov 2018 Co-organizer for the *Fields Symposium in honor of Maryam Mirzakhani*, Fields Institute, Toronto.
- 2018-present Associate Editor for *The Journal of Iranian Mathematical Society*
- 2016-present Associate Editor for *The Journal of the Ramanujan Mathematical Society*
- 2015-present Mentor for Association for Women in Mathematics (AWM) Mentor Network

## Collaborators

- Yael Algom-Kfir (University of Haifa)
- Mladen Bestvina (University of Utah)
- Mathew Clay (University of Arkansas)
- Young-Eun Choi
- Moon Duchin (Tufts University)
- Alex Eskin (University of Chicago)
- W Patrick Hooper (CUNY City College)
- Fanny Kassel (IHES)
- Misha Kapovich (Univ. of California Davis)
- Anna Lenzhen (Rennes)
- Howard Masur (University of Chicago)
- Maryam Mirzakhani (Stanford University)
- Yulan Qing (University of Toronto)
- Nikita Selinger (University of Alabama)
- Caroline Series (University of Warwick)
- Jing Tao (University of Oklahoma)
- Michael Yampolsky (University of Toronto)
- Juliette Bavard (Rennes)
- Jeffery Brock (Yale University)
- Camille Horbez (Orsay)
- Spencer Dowdall (Vanderbilt University)
- David Dumas (Univ. of Illinois at Chicago)
- Maxime Fortier Bourque (Univ. of Glasgow)
- Eriko Hironaka (Florida State University)
- Ilya Kapovich (CUNY Hunter College)
- Christopher J. Leininger (Rice University)
- Kathryn Mann (Cornell University)
- Babak Modami (SUNY Stony Brook)
- Mahan Mj (Tata Institute)
- Anja Randecker (University of Toronto)
- Saul Schleimer (University of Warwick)
- Juan Souto (Rennes)
- Yvon Verberne (University of Toronto)