University of Toronto
Department of Mathematics

MAT332H1F, Graph Theory

Midterm, October 24, 2017

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First	Last	Student Number

Instructions: No aids allowed. Write solutions on the space provided. To receive full credit you must show all your work. If you run out of room for an answer, continue on the back of the page. This exam has 7 questions, for a total of 50 points.

Problem #	Grade
1	
2	
3	
4	
5	
Bonus	
Total	

1. (8 points) Define the following terms and expressions:

   (a) Eulerian circuit

   (b) Induced subgraph

   (c) Complete graph

   (d) Maximal matching

2. (12 points) Answer true of false. Justify your answer with an argument or a counter example.

(a) ☐ A minimal vertex cover is always a minimum vertex cover.

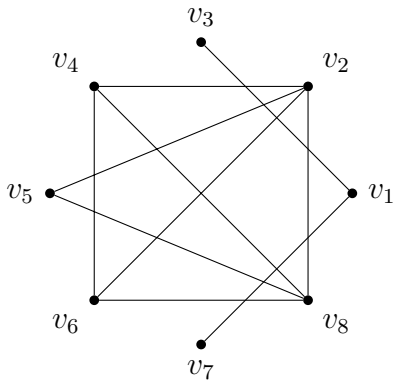
(b) ☐ The sum of the degrees of the vertices of a graph is always even.

(c) ☐ All spanning trees of a graph G have the same number of edges.

(d) ☐ Every complete bi-partite graph is vertex transitive.

3. (10 points) State and prove Cayley?s Formula.

4. (10 points) Let G be the following graph.



(a) What are the connected components of G?

(b) Find a spanning tree for each connected component of G.

(c) Is G bi-partite? Why?

(d) Does G have a cut vertex?

5. (10 points) A tournament of 10 teams lasted for 9 days, as follows: On each day, every team played one game against another team, with one team winning and one team losing in each of the 5 games. Over the course of the tournament, each team played every other team exactly once. Can one necessarily choose one winning team from each day without choosing any team more than once? (Hint: Use Hall's theorem.)

6. (Bonus Problem) Prove that if the diameter of a graph is at least 3, then the radius of its complement is at most 2.

7. (Bonus Problem) Draw a picture of yourself taking the test!