POISSON GEOMETRY AND THE KASHIWARA-VERGNE CONJECTURE

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Abstract. We give a Poisson-geometric proof of the Kashiwara-Vergne conjecture for quadratic Lie algebras, based on the equivariant Moser trick.

Geometrie de Poisson et la conjecture de Kashiwara-Vergne

Résumé. Dans cette note nous presentons une demonstation de la conjecture de Kashiwara-Vergne pour les algèbres de Lie quadratiques en utilisant des idées de la geometrie de Poisson et en particulier le truc de Moser equivariant.

1. Introduction

In 1977 Duflo [8] established the local solvability of bi-invariant differential operators on arbitrary finite-dimensional Lie groups G. In a subsequent paper, Kashiwara-Vergne [9] conjectured a property of the Campbell-Hausdorff series that would imply Duflo's result as well as a more general statement on convolution of (germs of) invariant distributions. They proved this property (cf. Section 2 below) for solvable Lie algebras.

Except for case $\mathfrak{g} = \mathfrak{sl}(2, \mathbb{R})$ proved by Rouvière [10], there had not been much progress on the conjecture for more than twenty years until Vergne [12] settled the case of quadratic Lie algebras. Shortly later, results for the general case were obtained in [6, 5, 11] using Kontsevich's approach to deformation quantization.

In this note we will give a new proof of the Kashiwara-Vergne conjecture for quadratic Lie algebras, using ideas from Poisson geometry. We will introduce a family of Poisson structures P_t on a neighborhood of the origin in $\mathfrak{g} \times \mathfrak{g}$, with the property that the diagonal action of G is Hamiltonian, with moment map $\Phi_t(X, Y) = \frac{1}{t}\log(e^{tX}e^{tY})$. Using Moser's argument we will define a time dependent vector field v_t interpolating the Poisson structures and moment maps, and in particular relating addition $\Phi_0 = X + Y$ and multiplication $\Phi_1 = \log(e^X e^Y)$. We will see that Moser's vector field directly gives a solution to the Kashiwara-Vergne problem.

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2. Geometric formulation of the Kashiwara-Vergne property

Let $\mathfrak{g}$ be a finite dimensional Lie algebra, and G the corresponding simply connected Lie group. Choose an open neighborhood $U \subset \mathfrak{g}$ of 0, such that the exponential map $\exp : \mathfrak{g} \to G,\ X \mapsto e^X$ is a diffeomorphism over U. Denote by $\log : \exp(U) \to \mathfrak{g}$ the inverse. Let $V \subset \mathfrak{g} \times \mathfrak{g}$ be a convex open neighborhood of the origin, with the property that $e^X e^Y \in \exp(U)$ for $(X, Y) \in V$. Given $F : V \to \mathfrak{g}$ denote by $F\delta_X$ the vector field $f \mapsto \frac{d}{ds}|_{s=0} f(X + sF(X, Y), Y),\quad f \in C^\infty(V)$, and by $\delta_X F$ the $\mathrm{End}(\mathfrak{g})$-valued function, $\quad a \mapsto \frac{d}{ds}|_{s=0} F(X + sa, Y),\quad a \in \mathfrak{g}$. Similarly, define $F\delta_Y$ and $\delta_Y F$.

Conjecture (Kashiwara-Vergne) There exist $\mathfrak{g}$-valued analytic functions A, B on a neighborhood of the origin of $\mathfrak{g}\times\mathfrak{g}$, with $A(0,0)=B(0,0)=0$, such that

$$\log(e^Y e^X) - X - Y = (1-e^{-\operatorname{ad}_X})A(X,Y) + (e^{\operatorname{ad}_Y}-1)B(X,Y) \tag{1}$$

and

$$\operatorname{tr}(\operatorname{ad}_X \delta_X(A) + \operatorname{ad}_Y \delta_Y(B)) = -\frac{1}{2}\operatorname{tr}\left(\frac{\operatorname{ad}_X}{e^{\operatorname{ad}_X}-1} + \frac{\operatorname{ad}_Y}{e^{\operatorname{ad}_Y}-1} - \frac{\operatorname{ad}_Z}{e^{\operatorname{ad}_Z}-1} - 1\right) \tag{2}$$

where $Z = \log(e^X e^Y)$.

Kashiwara-Vergne arrived at these conditions from simple and natural geometric considerations. They considered a deformation of the Lie bracket $[\cdot,\cdot]_t = t[\cdot,\cdot]$ and examined the limit $t\to 0$ where the Lie algebra becomes Abelian. Retracing their argument, one has the following equivalent formulation of the Conjecture. Consider the vector field v and the $\mathfrak{g}$-valued function Φ on V given by

$$v = (\operatorname{ad}_X A)\delta_X + (\operatorname{ad}_Y B)\delta_Y, \quad \Phi(X,Y) = \log(e^X e^Y). \tag{3}$$

Let $m_t : V \to V$ denote multiplication by $t\in[0,1]$, and rescale $v_t = \frac{1}{t}m_t^* v$ and $\Phi_t = \frac{1}{t}m_t^*\Phi$. (Note that this is well-defined even for $t=0$, and that $\Phi_0(X,Y) = X+Y$.) Let $J(X) = \det\left(\frac{1-e^{-\operatorname{ad}_X}}{\operatorname{ad}_X}\right)$ be the Jacobian of the exponential map, and denote by $\kappa_t \in C^\infty(V)$ the combination,

$$\kappa_t(X,Y) = \frac{J^{1/2}(tX)J^{1/2}(tY)}{J^{1/2}(t\Phi_t)},$$

Let Γ denote a translation invariant volume form on $\mathfrak{g}\times\mathfrak{g}$.

Proposition 2.1 (Kashiwara-Vergne). *Suppose $\mathfrak{g}$ is unimodular. Then (1), (2) are respectively equivalent to the statements that the vector field v_t intertwines the maps Φ_t and the volume forms $\kappa_t\Gamma$:*

$$(\frac{\partial}{\partial t} + v_t)\Phi_t = 0, \quad (\frac{\partial}{\partial t} + v_t)(\kappa_t\Gamma) = 0. \tag{4}$$

For the proof, see [9], p.255–257. The unimodularity assumption enters the reformulation of (2), since the left hand side is the divergence of v in this case.

3. Poisson geometry

Our approach to the Kashiwara-Vergne conjecture requires some elementary concepts from Poisson geometry, which we briefly review in this Section.

3.1. **Basic definitions.** A Poisson manifold is a manifold M, equipped with a bi-vector field $P\in C^\infty(M,\wedge^2 TM)$ such that the Schouten bracket $[P,P]$ vanishes. Let $P^\sharp : T^*M \to TM$ be the bundle map defined by P, i.e. $P^\sharp(a) = P(a,\cdot)$ for all covectors a. The Poisson structure gives rise to a generalized foliation of M, such that the tangent bundle to any leaf $L\subset M$ equals $P^\sharp(T^*M)|_L$. Each leaf carries a symplectic form ω_L given as the inverse of $P|_L$. Let Λ_L denote the symplectic volume form on L.

For any function $H\in C^\infty(M)$ one defines the Hamiltonian vector field $v_H = -P^\sharp(\mathrm{d}H)$. An action of a Lie group G on M is called Hamiltonian if there exists an equivariant *moment* *map* $\Phi \in C^\infty(M, \mathfrak{g}^*)$ with $X_M = -v_{\langle \Phi, X\rangle}$, where $X_M = \frac{d}{dt}|_{t=0} \exp(-tX)$ is the vector field generated by $X \in \mathfrak{g}$. Equivalently, the restriction of Φ to any symplectic leaf is a moment map in the sense of symplectic geometry. For any Lie group G, the dual of the Lie algebra $\mathfrak{g}^*$ carries a unique *Kirillov Poisson structure* such that the identity map is a moment map for the co-adjoint action. Its symplectic leafs are the co-adjoint orbits.

For any volume form Γ on a Poisson manifold (M, P), one defines the *modular vector field* by $w_\Gamma(H) = -\operatorname{div}_\Gamma(v_H)$. The modular vector field for $M = \mathfrak{g}^*$ is given by modular character: That is, $w = \sum_a \operatorname{tr}(\operatorname{ad}(e_a)) \frac{\partial}{\partial \mu_a}$ where e_a is a basis of $\mathfrak{g}$ and μ_a the dual coordinates on $\mathfrak{g}^*$. In particular, $w = 0$ for unimodular Lie algebras.

Lemma 3.1. *Let M be a Poisson manifold and Γ be a volume form on M. Suppose that the modular vector field vanishes. Then, for any vector field v of the form $v = \sum_i F_i v_{H_i}$, with smooth functions F_i, H_i, one has*

$$\operatorname{div}_\Gamma(v)|_L = \operatorname{div}_{\Lambda_L}(v|_L).$$

for all symplectic leafs L.

Proof. We have $\operatorname{div}_{\Lambda_L}(v_{H_i}|_L) = 0$ since Hamiltonian vector fields preserve the symplectic form, and we have $\operatorname{div}_\Gamma(v_{H_i})|_L = 0$ since the modular vector field vanishes. Hence, both sides equal the restriction of $\sum_i v_{H_i}(F_i)$ to L. □

3.2. **Changing the Poisson structure.** There is an easy way of constructing new Hamiltonian Poisson structures out of given ones. It will be convenient to state this using the language of equivariant de Rham theory. Let M be a G-manifold, where G is a connected Lie group. One defines a complex of equivariant differential forms $(\Omega_G^\bullet(M), \mathrm{d}_G)$, where $\Omega_G(M)$ is the space of polynomial maps $\alpha : \mathfrak{g} \to \Omega(M)$ satisfying the equivariance condition, $\alpha([Y, X]) + L_{Y_M}\alpha(X) = 0$. The equivariant differential is defined as $(\mathrm{d}_G\alpha)(X) = \mathrm{d}\alpha(X) - \iota_{X_M}\alpha(X)$. One introduces a grading on $\Omega_G(M)$ by declaring that $\deg(\alpha) = 2k + l$ if $X \mapsto \alpha(X)$ is a homogeneous polynomial of degree k with values in $\Omega^l(M)$. In particular, equivariant 2-forms are sums $\sigma_G(\xi) = \sigma + \langle \Psi, \xi\rangle$ where σ is an invariant 2-form and Ψ an equivariant map into $\mathfrak{g}^*$. Note that these definitions extend to *local* G-actions (equivalently, Lie algebra actions).

Suppose (M, P_0, Φ_0) is a Hamiltonian Poisson G-manifold, and $\sigma_G = \sigma + \Psi \in \Omega_G^2(M)$ an equivariant cocycle. Let $\sigma^\flat : TM \to T^*M$ be the bundle map defined by σ. Assume that $\det(1 + \sigma^\flat \circ P_0^\sharp) > 0$ everywhere. Then there is a well-defined bivector field P_1 on M such that

$$P_1^\sharp = P_0^\sharp \circ (1 + \sigma^\flat \circ P_0^\sharp)^{-1}. \tag{5}$$

Let $\Phi_1 = \Phi_0 - \Psi$. Then (M, P_1, Φ_1) is again a Hamiltonian Poisson G-manifold, with the same symplectic leafs as P_0. For any leaf L, the equivariant symplectic forms $(\omega_j)_G = \omega_j - \Phi_j|_L$ are related by $(\omega_1)_G = (\omega_0)_G + \iota_L^*\sigma_G$, and the volume forms by $\Lambda_{L,1} = \kappa|_L \Lambda_{L,0}$ where $\kappa = \det^{1/2}(1 + \sigma^\flat \circ P_0^\sharp)$.

3.3. **Moser's trick for Poisson manifolds.** Let M be a G-manifold, with a family of Hamiltonian Poisson structures (P_t, Φ_t) depending smoothly on t, where the symplectic foliation is independent of t. Suppose there exists a family of invariant 1-forms $\alpha_t \in \Omega^1(M)^G$, depending smoothly on t, such that for every symplectic leaf $\iota_L : L \hookrightarrow M$,

$$d_G(\iota_L^*\alpha_t) = \frac{d(\omega_t)_G}{dt}.$$

We will call $v_t = P_t^\sharp(\alpha_t)$ the *Moser vector field.* Restricting to symplectic leafs, the usual equivariant Moser's trick from symplectic geometry (cf. [7, Theorem 7.3] or [3, Lemma 3.4]) shows that v_t intertwines the Poisson structures and moment maps:

$$(\frac{\partial}{\partial t}+v_t)\Phi_t=0,\ \ (\frac{\partial}{\partial t}+v_t)P_t=0.$$

4. Proof of the Kashiwara-Vergne conjecture for quadratic $\mathfrak{g}$

4.1. **Deformation of the Kirillov Poisson structure.** Suppose $\mathfrak{g}$ is a quadratic Lie algebra. That is, $\mathfrak{g}$ comes equipped with a non-degenerate invariant symmetric bilinear form $\cdot$, used to identify $\mathfrak{g}^*\cong\mathfrak{g}$. Quadratic Lie algebras are unimodular. Let P_0 denote the Poisson structure on $\mathfrak{g}\times\mathfrak{g}$ given as a product of Kirillov Poisson structures. The moment map for the diagonal G-action is the sum $\Phi_0(X,Y)=X+Y$. Our goal is to find an equivariantly closed equivariant 2-form $\sigma_G=\sigma+\Psi\in\Omega^2_G(V)$ such that $\Psi=\Phi_0-\Phi_1$, where $\Phi_1(X,Y)=\log(e^Xe^Y)$. The construction is motivated by ideas from the theory of group-valued moment maps, introduced in [1]. Let $\theta^L,\theta^R\in\Omega^1(G,\mathfrak{g})$ denote the left/right invariant Maurer-Cartan on G. Let G act on itself by conjugation, and let $\eta_G\in\Omega^3_G(G)$ be the equivariant Cartan 3-form

$$\eta_G(X)=\frac{1}{12}\theta^L\cdot[\theta^L,\theta^L]-\frac{1}{2}(\theta^L+\theta^R)\cdot X$$

It is well-known that $\mathrm{d}_G\eta_G=0$. One easily calculates the pull-back of this form under group multiplication $\mathrm{Mult}:G\times G\to G$:

$$\mathrm{Mult}^*\eta_G=\eta_G^1+\eta_G^2-\frac{1}{2}\mathrm{d}_G(\theta^{L,1}\cdot\theta^{R,2}) \tag{6}$$

where the superscripts $1,2$ denote pull-back to the respective G-factor.

Define an equivariant 2-form $\varpi_G\in\Omega^2_G(\mathfrak{g})$ by applying the de Rham homotopy operator for the vector space $\mathfrak{g}$ to $\exp^*\eta_G\in\Omega^3_G(\mathfrak{g})$. The form degree 0 part of ϖ_G is minus the identity map $\mathfrak{g}\to\mathfrak{g}$, $Y\mapsto Y$, so that $\varpi_G(X)=\varpi-Y\cdot X$ at $Y\in\mathfrak{g}$. (See [2, Section 6.2] or [12, Section 2] for an explicit formula for the form degree 2 part ϖ.) We now define $\sigma_G=\sigma+\Psi\in\Omega^2_G(V)$ by

$$\sigma_G=\Phi_1^*\varpi_G-\varpi_G^1-\varpi_G^2+\frac{1}{2}(\exp^*\theta^L)^1\cdot(\exp^*\theta^R)^2.$$

Notice that $\Psi=\Phi_1-\Phi_0$ as desired, and $\mathrm{d}_G\sigma_G=0$ by (6). Since P_0 vanishes at the origin, it follows (choosing V smaller if necessary) that $1+\sigma^\flat\circ P_0^\sharp$ is invertible over V. Hence (5) defines a new G-equivariant Poisson structure P_1 on V, with moment map $\Phi_1(X,Y)=\log(e^Xe^Y)$.

We now scale the Poisson structure on V, by setting $P_t:=t\,m_t^*P_1$ for $0<t\le 1$, with moment map Φ_t. Since $t\,m_t^*P_0=P_0$, we see that

$$P_t^\sharp=P_0^\sharp(1+\sigma_t^\flat\circ P_0^\sharp)^{-1},$$

where $\sigma_t=t^{-1}m_t^*\sigma$. Since σ is a 2-form, $\lim_{t\to 0}\sigma_t=0$, and therefore $P_0=\lim_{t\to0}P_t$. Let $\alpha_t=\frac{1}{t^2}m_t^*\alpha_1\in\Omega^1(V)$ be the family of 1-forms obtained by applying the de Rham homotopy operator to the closed 2-forms $\frac{d\sigma_t}{dt}$. Since $P_t=tm_t^*P_1$, the Moser vector field $v_t=P_t^\sharp(\alpha_t)$ scales according to $v_t=\frac{1}{t}m_t^*v_1$. Note that α_t vanishes at the origin, as does any form in the image of the homotopy operator.

To write v_1 in the form (3), define $\mathfrak{g}$-valued functions A, B on V, vanishing at the origin, by

$$(1+\sigma_1^\flat \circ P_0^\sharp)^{-1}\alpha_1 = A\,\mathrm{d}X + B\,\mathrm{d}Y.$$

Then $P_0^\sharp(A\mathrm{d}X + B\mathrm{d}Y) = v_1$, showing that v_1 is given by (3).

4.2. **A solution to the Kashiwara-Vergne problem.** We now show that the vector field $v = v_1$ provides a solution to the Kashiwara-Vergne conjecture in the form (4). The first condition $(\frac{\partial}{\partial t} + v_t)\Phi_t = 0$ is automatic since v_t is a Moser vector field. By Lemma 3.1, and since the modular vector field for the Kirillov Poisson structure of a unimodular Lie algebra vanishes, the second condition $(\frac{\partial}{\partial t} + v_t)\kappa_t\Gamma = 0$ holds if and only if v_t intertwines the volume forms $\kappa_t|_{\mathcal{O}}\Lambda_{\mathcal{O}}$ on products of orbits $\mathcal{O} = \mathcal{O}_1 \times \mathcal{O}_2$, where $\Lambda_{\mathcal{O}}$ is the symplectic volume form on $\mathcal{O}$. By the following Proposition, the Moser vector field v_t does indeed have this property.

Proposition 4.1. *For any product of orbits $\mathcal{O} = \mathcal{O}_1 \times \mathcal{O}_2 \subset V$, the volume form $\kappa_t|_{\mathcal{O}}\Lambda_{\mathcal{O}}$ is the Liouville form of $\mathcal{O}$ with respect to the Poisson structure P_t.*

Letting $\omega_{\mathcal{O}}$ denote the symplectic form on $\mathcal{O}$, this Lemma says that the top form degree part of $\exp(\omega_{\mathcal{O}} + \iota_{\mathcal{O}}^*\sigma_t)$ equals the top form degree part of $\kappa_t|_{\mathcal{O}}\exp(\omega_{\mathcal{O}})$. It is possible, but tedious, to prove this by a direct check of the equality $\kappa_t = \det^{1/2}(1+\sigma_t^\flat \circ P_0^\sharp)$. An alternative route uses the theory of group-valued moment maps [1]. We briefly summarize the features of this theory needed here:

(i) Suppose M is a G-manifold, and $\omega_G = \omega - \Phi$ an equivariant cocycle with ω non-degenerate. Let Λ be the Liouville form defined as the top form degree part of $\exp\omega$. Let $\tilde{\omega} = \omega_G - \Phi^*\varpi_G$, and $\tilde{\Phi} = \exp\circ\Phi : M \to G$. Then $\tilde{\omega}$ is non-degenerate near $\Phi^{-1}(0)$, and $\tilde{\Phi}$ satisfies the moment map condition for group-valued moment maps, $\mathrm{d}_G\tilde{\omega} + \tilde{\Phi}^*\eta_G = 0$. Let $\tilde{\Lambda}$ be the top form degree part of $\exp(\tilde{\omega})$, divided by the function $\det^{1/2}(\frac{1}{2}(\mathrm{Ad}_\Phi + 1))$. Then

$$\tilde{\Lambda} = (\Phi^* J^{1/2})\,\Lambda.$$

(ii) Suppose M_1, M_2 are two G-manifolds, equipped with equivariant maps $\tilde{\Phi}_j : M_j \to G$ and invariant 2-forms $\tilde{\omega}_j$ satisfying $\mathrm{d}_G\tilde{\omega}_j + \tilde{\Phi}_j^*\eta_G = 0$. Suppose for simplicity that $\tilde{\omega}_j$ are non-degenerate. Let $\tilde{\Lambda}_j$ be defined as in the last paragraph. Let $M_1 \times M_2$ be equipped with the diagonal G-action, group-valued moment map $\tilde{\Phi} = \tilde{\Phi}_1\tilde{\Phi}_2$, and 2-form

$$\tilde{\omega} = \tilde{\omega}_1 + \tilde{\omega}_2 + \frac{1}{2}\tilde{\Phi}_1^*\theta^L \cdot \tilde{\Phi}_2^*\theta^R.$$

Then $\mathrm{d}_G\tilde{\omega} + \tilde{\Phi}^*\eta_G = 0$, and the volume form $\tilde{\Lambda}$ constructed from $\tilde{\omega}, \tilde{\Phi}$ is simply the direct product $\tilde{\Lambda}_1 \times \tilde{\Lambda}_2$.

These facts are proved in [4]. (The results in that paper were stated for $\mathfrak{g}$ is compact, but the proofs work for quadratic $\mathfrak{g}$, with obvious modifications.) In our case, we start out with two coadjoint orbits $\mathcal{O}_j$, with moment maps Φ_j the inclusion into $\mathfrak{g}$. Let Λ_j be the Liouville forms. "Exponentiating" as in (i), the group-valued moment maps $\tilde{\Phi}_j$ are $\exp\circ\Phi_j$, and the volume forms become $J^{1/2}\Lambda_j$. Taking the product as in (ii), $\mathcal{O}_1 \times \mathcal{O}_2$ becomes a space with group-valued moment map $\tilde{\Phi}(X,Y) \mapsto e^X\,e^Y$ and volume form $J^{1/2}(X)J^{1/2}(Y)\Lambda$ where $\Lambda = \Lambda_1 \times \Lambda_2$. Using (i) in reverse, $\mathcal{O}_1 \times \mathcal{O}_2$ becomes a Hamiltonian G-space in the usual sense, with moment map $\Phi(X,Y) = \log(e^X e^Y)$ and equivariant symplectic form $\omega_G + (\Phi_1, \Phi_2)^*\sigma_G$. The Liouville volume form is $\frac{J^{1/2}(X)J^{1/2}(Y)}{J^{1/2}(Z)}\Lambda$. This yields a proof of Proposition 4.1 for $t = 1$, the general case follows by rescaling the given bilinear form on $\mathfrak{g}$.

Remark 4.2. Our solution to the Kashiwara-Vergne problem for quadratic Lie algebras differs from Vergne's solution in [12], although the ingredients are very similar. A common feature of both solutions is that they are, in fact, independent of the quadratic form.

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