APM 426H/ MAT 1700H Prof. Robert McCann www.math.utoronto.ca/mccann/

GENERAL RELATIVITY

Lecture Hours: 13h10-15h00 Tuesdays OI 5170 and Thursdays 14h10-15h00 SS 1069
Office Hours: 15h15-16h00 Tuesdays in BA 6124 (or on Zoom by request)

Einstein's theory of gravity. Special relativity and the geometry of Lorentz manifolds. Gravity as a manifestation of spacetime curvature. Einstein's equations. Cosmological implications: big bang and inflationary universe. Schwarzschild stars: bending of light and perihelion precession of Mercury. Topics from black hole dynamics and gravitational waves. Penrose singularity theorem.

Prerequisite(s) for APM 426H1: MAT 363H1 Surfaces or 367H1 Differential Geometry

Required References: (for more recommendations see supplementary reference sheet)

Robert Wald. General Relativity, University of Chicago Press, 1984.
Bernhard Schutz. A First Course in General Relativity 3rd ed, Cambridge Press, 2022
(alternately, the 2nd edition of same is available online through UofT's library).

Course objectives: 1. To acquire an understanding and familiarity with Lorentzian geometry, which is the mathematical framework for Einstein's theory of gravity, general relativity.

2. To acquire a working knowledge of basic examples in this theory: linearized gravity and the gravitational waves it predicts, the homogeneous isotropic Friedman-LeMaitre-Robertson-Walker models for our universe, and the spherically symmetric Schwarzschild solution representing the spacetime geometry near a (nonrotating) star or black hole

3. To develop facility doing computations in such geometries, including solving for geodesics, various kinds of curvature, and exploiting symmetries (e.g. Killing vector fields).

4. To be able to interpret the physical implications of this theory, and derive quantitative and qualitative predictions, such as gravity waves, expansion of the universe, the time since big bang, local conservation of energy and momentum, and the perihelion precession of Mercury.

5. To develop your ability both to learn advanced mathematical and physical material on your own, and to communicate your findings clearly, succinctly, and effectively, orally and in writing.

Contact: The instructor can be contacted by email at `mccann@math.utoronto.ca`. Please enter APM 426 (or MAT 1700) in the subject line and allow 72 hours for a response. Inquiries about graded assignments should be directed first to TA Sabrina Lin `<srn.lin@mail.utoronto.ca>` and then escalated to the instructor only if necessary. Please write in a professional manner.

Schedule by week

1 Overview, Special Relativity and the Lorentz group
2 Manifolds and Vector Fields
3 Riemannian and Lorentzian Metrics; Tensor Fields
4 Perfect Fluids and Stress-Energy-Momentum Tensor
5 Geodesics: Covariant Derivatives and Christoffel Symbols
6 Riemannian and Ricci Curvature; Geodesic Deviation
* 17-21 February READING WEEK
7 The Einstein Equation and its Linearization (Newtonian Limit)
8 Gravity Waves; Killing Fields and Conserved Quantities
9 Cosmological Implications: The Expanding Universe and Big Bang
10 Black Holes & Stars: Schwarzschild Solution, Perihelion Precession and Deflection of Light
11 Causality and Singularity Theorems (as time permits)
12 Student Project Presentations

GRADING SCHEME

10 % Attendance and participation (can be substituted by oral exam if requested by 31 Jan 2025)
40 % Weekly problem sets, turned in on Crowdmark (or possibly an alternate platform)
35 % Written project, turned in on Crowdmark AND by hardcopy in class
15 % Brief oral presentation (poster or laptop based) of project in class Apr 1 (or possibly Apr 3).

At the discretion of the instructor, submissions may be accepted up to a maximum of three days late. If accepted, they will be penalized by 5% per day late.

PROJECT DESCRIPTION AND DEADLINES

10% Wednesday 12 Feb: Settle on a topic and sources and communicate them to me by email
90% Monday 24 March: Final written report due. You must subsequently provide the poster or laptop presentation accompanying your oral report to me immediately following your presentation (i.e. Apr 1, unless you are an individual whom I have asked in advance to present on Apr 3 instead).

This project is intended to provide you with an opportunity to independently pursue some topic in general relativity which interests you, within the framework of the course. Each of you will be expected to prepare both a written report on your topic (something on the order of 6-10 pages), and a short (likely 8 minute) presentation (laptop or poster, as you prefer) based on it. The goal of your presentations should be to communicate as much mathematical and physical understanding and intuition for your topic as possible in the given space and time constraints.

For topics, I suggest reading Kip Thorne's book "Black Holes and Time Warps' to find an aspect of relativity of particular interest to you. One format that your project could take would be to make rigorous sense of a few of Thorne's claims by finding the equations behind the intuitive scenarios that he describes. Another possibility would be to review abstracts for the presentations at the Fall 2022 Fields Thematic semester organized by the instructor

`www.fields.utoronto.ca/activities/22-23/nonsmooth-relativity`
`http://www.fields.utoronto.ca/activities/22-23/nonsmooth-regularity`
or at this ongoing online colloquium series
`https://jomarec.org/`

and watch a few of the recorded lectures to get some exposure to topics of current research. Your project doesn't need to take this form, but in any case you should settle on a topic, format, and references in consultation with me before 12 Feb. I can also suggest some topics: Why is gravity purely attractive? (the positive mass theorem); a least action principle for Einstein's equations; disk scheduling & airline boarding via Lorentz geometry, regularity of optimal maps, singularity theorems, splitting theorems, nonsmooth formulations of the Einstein equation.

Here are some further guidelines on projects. Your project should have a bibliography with complete bibliographic data: publisher and city for books; journal, volume and page numbers for articles; years for both; URL's for web references. Articles and books which have been peer-reviewed are generally more reliable than sources which have not, so at least two of your sources should be peer-reviewed. Formulas, including displayed equations, should be punctuated as part of the sentences in which they appear.

This course is governed by University of Toronto policies, including the following:

Academic Integrity

You are expected to work on your project by yourself. All the sources you use should be credited appropriately in a bibliography at the end. I encourage you to have a complete draft by the middle of March, and to solicit feedback on it from a classmate, a peer, or any of the writing centers on campus to help you improve grammar, typos, flow, format and the quality of your

exposition. Similarly, you should rehearse your oral presentation at least a week in advance with some classmates or friends, to work out the level and pace and give yourself time to revise and make adjustments.

A plagiarism detection tool may be used for detecting plagiarism in some of the written work submitted in this course. Normally, students will be required to submit their written work to the universitys plagiarism detection tool for a review of textual similarity and detection of possible plagiarism. In doing so, students will allow their work to be included as source documents in the tools reference database, where they will be used solely for the purpose of detecting plagiarism. The terms that apply to the Universitys use of the this tools service are described on the Centre for Teaching Support and Innovation web site: *https://teaching.utoronto.ca/resources/plagiarism-detection/*

For problem sets, you may discuss the problems with other students in the class, but must write up your own solution in your own words. However, I recommend making a serious attempt at solving each problem set on your own before consulting classmates, and relying on classmates only insofar as is necessary, as you will develop your own skills and learn more that way. In the long run, a hint from a classmate will help you much more than a detailed solution ever will. If you consult additional resources (classmates, printed material other than the required texts, websites) you should acknowledge these in your submission and make sure you truly understand the concepts before writing up your solution. If you find yourself writing your problem set with a website open in front of you, you have not reached an appropriate level of understanding. So while you are welcome to work with classmates on problems that stymie you in the discovery phase, when it comes time to write up your solutions for submission, you must present solutions in your own words, i.e. do not work together when you are writing your solutions, credit your sources, never share your solutions with your peers, and never have someone else's solution in front of you.

All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters. If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, please reach out to me. Note that you are expected to seek out additional information on academic integrity from me or from other institutional resources (for example, the University of Toronto website on Academic Integrity: `http://academicintegrity.utoronto.ca/`

Equity, Diversity and Inclusion

The University of Toronto is committed to equity, human rights and respect for diversity. All members of the learning environment in this course should strive to create an atmosphere of mutual respect where all members of our community can express themselves, engage with each other, and respect one another's differences. U of T does not condone discrimination or harassment against any persons or communities.

Recordings of Lectures

If University Policy mandates a switch from in person to hybrid or online learning, then lectures will be recorded on Zoom and I may choose to make them available to students in the course asynchronously. If you do not wish to appear in the recording, please switch off your camera before asking a question. Course videos and materials belong to your instructor and/or the University depending on the specific facts of each situation and are protected by copyright. In this course, you are permitted to download session videos and materials for your own academic use, but you should not copy, share, or use them for any other purpose without the explicit permission of the instructor. For questions about the recording and use of videos in which you appear, please contact your instructor.

Accessibility

The University of Toronto provides academic accommodations for students with disabilities in

accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a collective obligation to develop an accessible learning environment that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs. Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require accommodations, please feel free to approach me and/or the Accessibility Services office, and register at `https://studentlife.utoronto.ca/department/accessibility-services/`

Other Academic and Personal Supports

- Writing Centre `https://writing.utoronto.ca/writing-centres/arts-and-science/`
- U of T Libraries `https://onesearch.library.utoronto.ca/`
- Student Code of Conduct: *https://governingcouncil.utoronto.ca/secretariat/policies/code-student-conduct-december-13-2019*
- Feeling Distressed? *https://studentlife.utoronto.ca/task/support-when-you-feel-distressed/*
- Academic Success Centre *https://studentlife.utoronto.ca/department/academic-success/*
- College/Faculty Registrars *https://future.utoronto.ca/current-students/registrars/*

Religious accommodation

The University provides reasonable accommodation of the needs of students who observe religious holy days other than those already accommodated by ordinary scheduling and statutory holidays. Students have a responsibility to alert members of the teaching staff in a timely fashion to upcoming religious observances and anticipated absences and instructors will make every reasonable effort to avoid scheduling tests, examinations or other compulsory activities at these times. Please reach out to me as early as possible to communicate any anticipated absences related to religious observances, and to discuss any possible related implications for course work.

Policy on missed term work

Students who are absent from class for prolonged periods and who require consideration for missed academic work should contact the instructor and verify their absence(s) through either the Absence Declaration tool, Verification of Illness or Injury (VOI) form, College Registrar Letter, or Letter of Academic Accommodation from Accessibility Services, as appropriate to their situation. `https://www.artsci.utoronto.ca/current/academics/student-absences`

The absence declaration can be used once per term. Outside of the one time absence declaration use, students must adhere to the alternate processes for absences linked above, as well as the missed work policy as set out above.

Re-marking Policy

A student who believes an individual item of work has been incorrectly or unfairly marked may ask the person who marked it for a re-evaluation. With evidence to back their appeal, students should make such requests as soon as reasonably possible after receiving the work back, but no later than 2 weeks after it was returned

Family care responsibilities

The University of Toronto strives to provide a family-friendly environment. You may wish to inform me if you are a student with family responsibilities. If you are a student parent or have family responsibilities, you also may wish to visit the Family Care Office website at `family-care.utoronto.ca`