William Vickrey: A Pioneer in the Economics of Incentives

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by

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It is with a lot of emotion that I will tell you a few words about William Vickrey's contribution to the prehistory of incentive theory, a field created by James Mirrlees in 1971.

W. Vickrey solved incentive problems as many others, very much like a French engineer-economist such as Jules Dupuit or Marcel Boiteux, i.e. with brilliant insights drawn from a deep knowledge of economic reality, with an unusual drive for concrete solutions, without necessarily a clear view of his contribution to the development of economic science.

In 1982, and it was the only time I met him, I gave a seminar on incentive theory at Columbia University. In the evening he did not talk about incentive theory, and as I was unaware of most of his contributions at the time, I did not ask him relevant questions on this topic. I remember only how sad he was to have spent 10 years trying to convince without success the N.Y. authorities of using his pricing proposals for the N.Y. subway.

What is incentive theory?

The economics of incentives can be described as the design of rules and institutions for inducing economic agents to exert high levels of effort (in a broad sense) and to reveal truthfully all socially relevant information they might have.

It is a step further beyond Jacob Marschak's (1955) theory of teams, which, as incentive theory, stresses the decentralization of information in multi-agent situations, but unlike game theory, neglects strategic behavior. From a theoretical perspective, incentive theory corresponds to a simple case of game theory with incomplete information in which bargaining has been eliminated by attributing to an economic agent, the principal, all the ability to design the rules of the games played by the other agents, under the only constraint that they must be induced to participate.

The principal suffers from asymmetric information and designs rules to maximize his expected utility, anticipating the strategic behavior of the agents for the rules he has proposed.

How can we briefly describe the scope of the results in incentive theory with reference to the principal-agents model I have just described. Three types of results can be distinguished.

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At one extreme we have the Gibbard-Satterthwaite (1973) impossibility theorem. Roughly speaking it shows that if he has no information on agents' preferences, very little can be achieved by the principal, with dominant strategy mechanisms, i.e. mechanisms for which each agent has a best strategy whatever the strategies of the other agents.

For example, if the principal is a constitutional designer who organizes the production of public goods, only dictatorial mechanisms which make no use of the other agents' information are possible.

At the other extreme, in the same example of public goods, if the principal knows that agents have quasi-linear utility functions–a very strong restriction on preferences–he can use the class of so called Clarke-Groves dominant strategy mechanisms, which achieve the first best up to some imbalance of the budget. (For quadratic utility functions the budget may even be balanced so that the exact first best can be achieved despite asymmetric information).

In between these two extremes, we have a myriad of second best problems in which the principal, who maximizes his expected utility under asymmetric information, makes a trade-off between efficiency and the allocation of informational rents to the agents.

The great achievement of incentive theory, through the work of J. Mirrlees, has then been to provide necessary and sufficient conditions describing the constraints imposed by the decentralization of information on the allocation of resources. By substituting incentive compatible feasibility to the classical notion of technical feasibility, incentive theory made one step further towards the real world, which required rewriting major chapters of economic theory.

W. Vickrey made remarkable contributions in these three directions 10 to 25 years before anyone else in the economics profession.

1. THE CONJECTURE OF GIBBARD-SATTERTHWAITE'S THEOREM (1960)

In his 1960 paper of the QJE, Vickrey provides an extremely lucid exposition of Arrow's theorem. Commenting upon welfare functions which do not satisfy the independence axiom, he discusses strategic misrepresentations of preferences. He then realizes the link between Arrow's assumptions and strategic manipulation.

p. 518:

> "It is clear that social welfare functions that satisfy the non-perversity and the independence postulates and are limited to rankings as arguments, are also immune to strategy".

Then comes the conjecture

p. 518:

> "It can be plausibly conjectured that the converse is also true, that is, that if a function is to be immune to strategy and be defined over a comprehensive range of admissible rankings, it must satisfy the independence criterion, though it is not quite so easy to provide a formal proof of this. Immunity to strategy and independence are thus at least closely similar requirements if not actually logically equivalent".

The route to the impossibility of non-manipulable non-dictatorial voting mechanisms through Arrow's theorem was suggested. A complete proof, the greatest achievement of social choice theory since Arrow's theorem, came thirteen years later. Quoting Gibbard.

> "Indeed the proof in this paper proceeds roughly by confirming Vickrey's conjecture".

2. THE DOMINANT STRATEGY MECHANISMS (1961)

Vickrey (1961) started from Lerner's suggestion (1944) that, where markets are imperfectly competitive, a state agency, through counterspeculation, "might be able to create the conditions whereby the marginal conditions for efficient resource allocation could be maintained". He then explored the possibility of organizing such mechanisms.

To impose the competitive price or determine the optimal allocation, "what the marketing agency needs... is an unbiased report of the marginal cost curves of the sellers and of the marginal value curves of the purchasers. The problem is then for the marketing agency to behave in such a way as to motivate the buyers and the sellers to furnish such unbiased reports. One method, though an expensive one, is to arrange to purchase the commodity from suppliers and to sell it to purchasers on terms that are dependent on the reported supply and demand curves in such a way that the suppliers and purchasers will maximize their profits, individually at least, by reporting correctly, so that any misrepresentation will subject them to risk of loss (or at least offer no prospect of gain)".

p.9.10 He proceeded by giving explicitly such a method. "The agency would first aggregate the reported supply and demand curves to determine the equilibrium marginal value, and apply this value to the individual demand and supply curves to determine the amounts to be supplied and purchased by the various individual buyers and sellers. The amount to be paid seller i, would however somewhat exceed the amount calculated by applying this marginal value to his amount supplied; in effect for the r^{th} unit supplied, seller i would have an amount equal to the equilibrium price that would have resulted if seller i had restricted his supply to r units, all other purchasers and sellers behaving competitively". Then follows a graphical proof that such a mechanism induces truthful revelation. These mechanisms were reinvented ten to twelve years later by Clarke (1971) and Groves (1973) in the context of public goods.

To see the connection think that the market price p is the choice of a public good with zero cost for which each trader has a willingness to pay equal to his net surplus, $V^i(p) = \int_0^p S^i(x)\,dx$ for a seller, $W^j(p) = \int_p^\infty D^j(x)\,dx$ for a buyer. However for these expressions to make sense p must be an equilibrium price, a restriction which does not exist in the usual public good problem.

The basic principle is then to give to the agent the value of the social externality he creates by his presence which is here, for example for a seller,

computed as follows: Let p^* the equilibrium price when all agents are present and p^{*-k} the equilibrium price when all agents except agent k are present. The externality is the difference between the social welfare of all agents except k for p^* and the same social welfare for p^{*-k}, i.e.:

$$\{\sum_{i\neq k} V^i(p^*) + \sum_j W^j(p^*)\} - \{\sum_{i\neq k} V^i(p^{*-k}) + \sum_j W^j(p^{*-k})\}$$
$$= \int_{p^*}^{p^{*-k}} \{\sum_j D^j(x) - \sum_{i\neq k} S^i(x)\}dx > 0 \qquad (1)$$

which is positive[1] because in this economy with no income effect, by his presence, the seller improves the welfare of the other participants to the market.

Let us derive graphically this expression. Social welfare of the agents except k is represented by the shaded area of Figure 1 when seller k is present in the economy and p^* is the resulting equilibrium price. Social welfare of the agents except k is represented by the shaded area of Figure 2 when seller k is absent in the economy and p^{*-k} is the equilibrium price. The positive externality created by seller k is the difference between these two shaded areas represented by the shaded area of Figure 3. It corresponds to the payment which has to be made to seller k to induce truthful revelation of his marginal cost curve. Repeating the exercise for each seller one gets the counterspeculative payments to suppliers obtained by Vickrey (1960) in his Figure 3.1, reproduced here as Figure 4, with a similar figure for buyers.

Vickrey was very sceptical about his mechanism because it required an external source of funds p. 13 "This solution would indeed permit optimum allocation of resources to be achieved if there were a source of public funds that was without adverse influence on resource allocation in other directions". He then conjectured the impossibility theorem derived in the seventies[2]: However, it seems that all modifications that do diminish the cost of the scheme either imply the use of some external information... or reintroduce a direct incentive for misrepresentation". He also noted the possibility of collusive behavior: "there remains under the scheme a positive incentive for firms to merge into larger units for the sake of obtaining more favorable treatment".

He did not see in this example that in fact this need for funds could be somewhat mitigated by adjusting constants in the transfers. Instead he turned to auctions where in fact the seller was going to be the external sink needed to achieve Pareto optimality.

The second price sealed bid auction he proposed was in fact just another version of the same mechanism in which the winner pays the externality he causes to the others, namely the second highest price. He showed that truthful bidding is a dominant strategy. To compare the first price and second

[1] In the case of a free public good this transfer is negative because the participation of an agent can only disturb the choice of public good of the other agents.

[2] See Green and Laffont (1979).

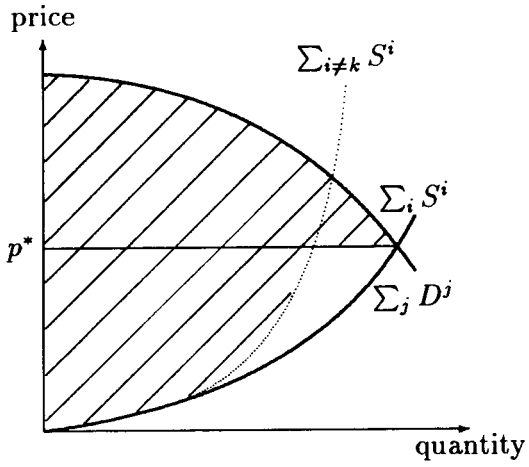

Figure 1

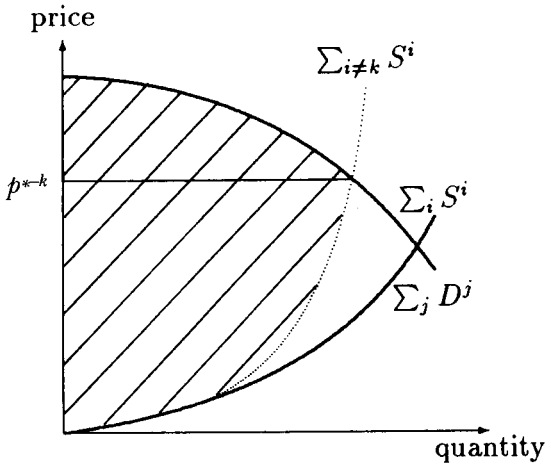

Figure 2

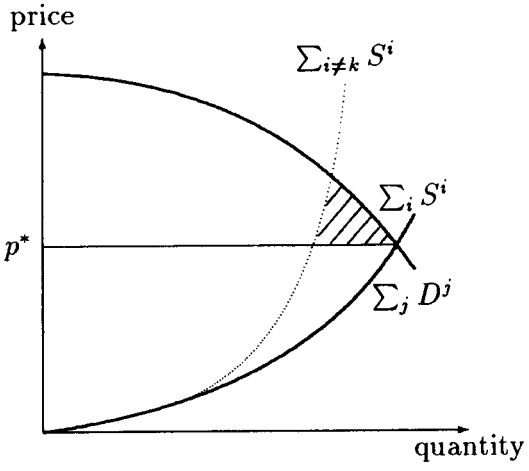

Figure 3

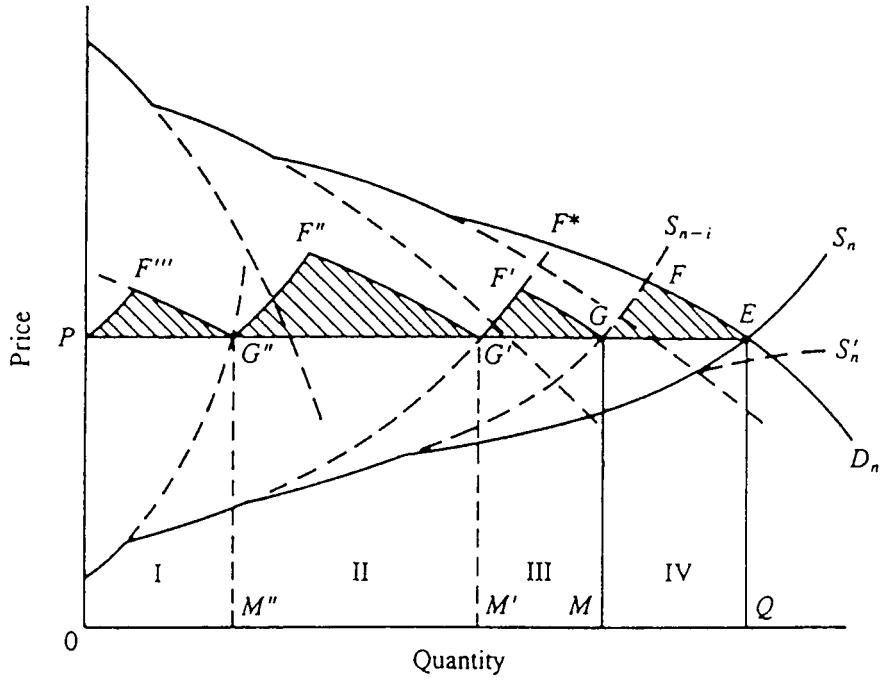

Figure 4. Counterspeculative Payments to Suppliers.

price auction and obtain a first version of the revenue equivalence theorem, he computed Bayesian equilibria (for particular distributions) many years before the formal definition of Harsanyi (1967–68) and explored the still quite unknown territories of asymmetric auctions and repeated auctions.

3. THE SECOND BEST OPTIMAL INCOME TAX PROBLEM

W. Vickrey had been senior economist of the tax research division of the US Treasury Department and tax expert of the Governor of Puerto Rico. His interest in incentives probably arose from his activities in these areas. As early as 1945, he used the insights of Von Neumann and Morgenstern (1994) to model the optimal income tax problem as a principal agent problem where the principal is the tax authority and the agents the tax payers.

Vickrey (1945) p. 329:

> "If utility is defined as that quantity the mathematical expression of which is maximized by an individual making choices involving risk, then to maximize the aggregate of such utility over the population is equivalent to choosing that distribution of income which such an individual would select were he asked which of various variants of the economy he would become a member of, assuming that once he selects a given economy with a given distribution of income he has an equal chance of landing in the shoes of each member of it".

Note in passing Harsanyi's (1955) interpretation of expected utility as a justice criterion.

With this objective function in mind, he formulates economically the problem of optimal income taxes.

"It is generally considered that if individual incomes were made substantially independent of individual effort, production would suffer and there would be less to divide among the population. Accordingly some degree of inequality is needed in order to provide the required incentives and stimuli to efficient cooperation of individuals in the production process".

"The question of the ideal distribution of income, and hence of the proper progression of the tax system, becomes a matter of compromise between equality and incentives".

He then proceeded to a formalization of the question as a calculus of variation problem but concluded that even in this simplified form the problem resists any facile solution.

The Pontryagin principle was far away and twenty six years were needed to obtain Mirrlees' 1971 solution.

Table 1. Prior information and incentive compatible mechanisms

NO INFORMATION $U(x,y)$	**VICKREY** **1960**	GIBBARD-SATTERTHWAITE 1963 1975 THEOREM DICTATORIAL MECHANISMS
ONE-DIMENSIONAL PARAMETRIC REPRESENTATION OF PREFERENCES $U(x,y,\theta)$ $U(.)$ known	**VICKREY** **1945**	SECOND BEST OPTIMAL MECHANISMS MIRRLEES 1971
QUASI-LINEAR PREFERENCES $y + u(x,\theta)$	**VICKREY** **1961**	CLARKE-GROVES 1971 1973 FIRST BEST BUT IMBALANCE OF THE BUDGET
QUADRATIC PREFERENCES LINEAR IN INCOME $y + \theta x - x^2$		GROVES-LOEB (1975) FIRST BEST

x: quantity of public good,
y: quantity of private good.

CONCLUSION

J. Marschak was probably the first economist aware of the generality of incentive problems that he chose not to deal with, despite his unique knowledge of the work of the statisticians in this area. In their own way statisticians also discovered incentive theory very early. Moral hazard was encountered in the sampling theory developed for quality control (Whittle (1954), Hill (1960)) and adverse selection was encountered in attempts to elicit the true probabilities of forecasters (Good (1952), McCarthy (1956)).

Marschak (1959) "The question raised by Good and McCarthy is extremely interesting, and, in fact opens up a new field of problems in the economics of information".

Marschak (1955) "This raises the problem of incentives. Organization rules can be devised in such a way that, if every member pursues his own goal, the goal of the organization is served... I shall have to leave the problem of incentives aside".

W. Vickrey, as I have tried to show, posed and solved incentive problems in the partial equilibrium contexts where it was the most fruitful to start the analysis (see table 1 for a summary).

The 1950's and 1960's were the great periods of general equilibrium theory and his work on incentives was totally ignored until the development of game theory under incomplete information and the informational revolution of the seventies.

A lot remains to be done to bring together these two main streams of economic science.

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