MAT244 Homework 6

due: December 9, 2022

1 (5 points)**.** Determine the matrix e^{Jt} for $J = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}$ and λ arbitrary.

2 (5 points)**.** Consider the n^{th}-order equation with constant coefficients

$$y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1 y' + a_0 y = 0.$$

(a) Let $x_i = y^{(i-1)}$ for all $i = 1, \ldots, n$. Find a matrix A such that $x' = Ax$.

(b) What is the characteristic polynomial of A?

(c) Let λ be an eigenvalue of A. What is the geometric multiplicity of λ?

3 (10 points)**.** Find parametric equations $(x(t), y(t))$ of a curve satisfying

$$\begin{aligned} x' &= 2x - y + \sin t \\ y' &= x + \cos t \end{aligned}$$

which (at $t = 0$) starts in the first quadrant and, after some time, leaves it forever. [*Note*: there are many answers—pick any one you like.]

4 (10 points)**.** Consider the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

(a) Plot the solution curve passing through the point $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(b) Express the curve in the form $y = F(x, y)$. [*Hint*: an integrating factor is $(x^2+y^2)^p$.]

5 (10 points)**.** Find the general solution to

$$x' = -\kappa \begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ & -1 & 1 & \\ & & \ddots & \ddots \end{pmatrix} x$$

where κ is a real constant and the blank entries are all 0. Your answer should involve infinitely many arbitrary constants $c_0, c_1, c_2, \ldots$