MAT244 Homework 4

due: November 4, 2022

1 (5 points). Which solutions of $y^{(5)} = y$ vanish as $x \to \infty$?

2 (10 points). Recall that $\cosh x = \frac{1}{2}(e^x + e^{-x})$. Solve

$$y''' - 4y'' + 5y' - 2y = 4x \cosh^2 x$$

by the method of undetermined coefficients.

3 (10 points). Solve

$$x^2 y'' + 3xy' + y = x^\alpha \ln x$$

for each real number α.

4 (10 points). Let $g(x)$ be a continuous function in a neighbourhood of $x = 0$. Find a particular solution of

$$y''' - y'' + y' - y = g(x)$$

in the form $y_p = \int_0^x f(x - t)g(t)\, dt$ for some continuous function f.

5 (5 points). Consider the equation

$$(D^2 - D + 1)y = g(x) = x^3 - 3x^2 + 1 \tag{1}$$

where D is the derivative operator.

(a) What is the least power of D that annihilates g?

(b) Formally expand

$$\frac{1}{1 - D + D^2}$$

as a power series in D, up to order equal to your answer to (a).

(c) Rearrange (1) and use your answer to (b) to obtain a candidate for y.

(d) Show that your answer to (c) really is a solution to (1).