Math 246S: Homework 6

Due at the beginning of tutorial Tuesday, March 6, 2012 at 8:10 PM sharp.

NOTE: In case of a TA strike, homework will only be graded after the strike.

(1) (a) Compute $(2+2i)^4$.
    (b) Solve the quadratic equation $z^2 + (1+i)z + i = 0$.
    (c) Find all the roots of $z^3 + i = 0$

(2) Let z_0 be a root of $z^n - 3 = 0$. Let $\xi_0, \cdots, \xi_{n-1}$ be the n-th roots of unity. Show that $z_0\xi_0, \cdots, z_0\xi_{n-1}$ give *all* the roots of $z^n - 3 = 0$.

(3) Let $p(z) := c_n z^n + \cdots + c_1 z + c_0$ be a polynomial. Show that if $|p(z)| \geq 1$ for all $z \in \mathbb{C}$ then $p(z)$ is a constant polynomial. Hint: the fundamental theorem of algebra might be useful.

(4) Prove the triangle inequality for complex numbers: $|z_1 + z_2| \leq |z_1| + |z_2|$ for all complex numbers z_1, z_2.

(5) Let $p(z)$ be a polynomial as above. Show that if $|p(z)| \leq 1$ for all $z \in \mathbb{C}$ then $p(z)$ is a constant polynomial.

(6) Let $p(z) := c_n z^n + \cdots + c_1 z + c_0$ be a polynomial with real coefficients, i.e. $c_i \in \mathbb{R}$.
    (a) Show that if z_0 is a root of $p(z)$, then the complex conjugate $\overline{z_0}$ is also a root of $p(z)$.
    (b) Find all the roots of the polynomial $p(z) = z^4 - 4z^3 + 4z^2 - 4z + 3$.