DEPARTMENT OF MATHEMATICS
UNIVERSITY OF TORONTO

Term Test 3 March 15,2005
MATH246Y
Examiners: J. Korman and P. Rosenthal
Duration: 90 minutes.

LAST NAME:_____

FIRST NAME:_____

STUDENT NUMBER:_____

- There are ten questions, each of which is worth 10 marks.
- This paper has a total of 11 pages, including this cover page.
- **No calculators, scrap paper, or other aids are permitted**.
- Write your answers in the space provided. Use the back sides of the pages for scrap works.
- **Do NOT tear any pages from this test.**

FOR MARKERS ONLY	
Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

1. Find the cardinality of

$$\{a + b\sqrt{7} \mid a, b \in \mathbb{Q}\}$$

and justify your answer.

2. Find the cardinality of the set of all points in $\mathbb{R}^3$ whose first coordinate is algebraic, and justify your answer.

3. Find the cardinality of the set of all numbers in $[0, 1]$ whose decimal expansion uses only the digits 0 and 7. Justify your answer.

4. Prove that the set of polynomials with rational coefficients is countable.

5. Find the cardinality of the set of all functions from $\mathbb{R}$ into $\mathbb{R}$. Justify your answer.

6. Find an explicit bijection (i.e. one-to-one, onto function) between the sets $\mathbb{R}$ and $\mathbb{R} \setminus \mathbb{N}$.

7. Find the cardinality of the set of all lines in the plane. Justify your answer.

8. Prove that the cardinality of the set of all finite subsets of the plane is c.

9. Let e be the real number which is the base of the natural logarithm function. For each of the following two sets, state whether or not it is a number field and justify your answer. Hint: you can use the fact that e is transcendental.
(a) $\{x + ye \mid x, y \in \mathbb{Q}\}$

(b) $\{x + ye \mid x, y \in \mathbb{R}\}$

10. Prove that the following set is a number field

$$\{a + b\sqrt{2} + c\sqrt{3} \mid a, b, c \in \mathbb{Q}\}.$$