DEPARTMENT OF MATHEMATICS
UNIVERSITY OF TORONTO

Term Test 1 October 19, 2004
MATH246Y
Examiners: J. Korman and P. Rosenthal
Duration: 90 minutes.

LAST NAME:____________________________

FIRST NAME:____________________________

STUDENT NUMBER:____________________________

- There are ten questions, each of which is worth 10 marks.
- This paper has a total of 11 pages, including this cover page.
- **No calculators, scrap paper, or other aids are permitted**.
- Write your answers in the space provided. Use the back sides of the pages for scrap works.
- **Do NOT tear any pages from this test.**

FOR MARKERS ONLY	
Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

1. Find a solution x to the congruence: $2x \equiv 7 \pmod{5}$.

2. Find the remainder when 3^{2463} is divided by 10.

3. Prove, by mathematical induction, that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

for all natural numbers n.

4. Suppose that S is a set of natural numbers with the following properties:
(a) $1 \in S$
(b) $k + 2 \in S$ whenever $k \in S$.
Prove that S contains all the odd natural numbers.
(Hint: you could use the well-ordering principle.)

5. Prove that $11^{n+2} + 12^{2n+1} \equiv 0 \pmod{133}$ for every natural number n.

6. (a) Prove that $x^2 \equiv 1 \pmod{p}$ implies $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$ for p a prime.

(b) Find an m and an x such that $x^2 \equiv 1 \pmod{m}$ and $x \not\equiv 1 \pmod{m}$ and $x \not\equiv -1 \pmod{m}$.

7. Prove that the only prime number p such that $(p-1)!+6$ is divisible by p, is $p=5$.

8. Find a solution x to the congruence $15x \equiv 4 \pmod{31}$.

9. Find the remainder when 30! is divided by 31.
(Remark: the remainder should be a nonnegative number less than 31.)

10. Suppose that l, m and n are natural numbers and p is a prime. Prove that

$$(l + m + n)^p \equiv l^p + m^p + n^p \pmod{p}.$$