MAT1312 Exercises 1

Due date: January 30, 2020)

(1) Show that the symplectic volume of a toric manifold is equal to the Euclidean volume of its moment polytope.

(2) Show that $\mathbb{C}P^2$ is the toric manifold whose moment polytope is the isosceles right triangle.

(3) Exhibit explicitly the subsets of $\mathbb{C}P^3$ for which the stabilizer under the standard action of $U(1)^3$ is

(a) isomorphic to $U(1)$

(b) isomorphic to $U(1)^2$

(c) isomorphic to $U(1)^3$.

Describe the images of these subsets under the moment map $\mu_{U(1)^3}$: show that they are

(a) facets (i.e. 2-dimensional faces)

(b) edges (i.e. 1-dimensional faces)

(c) vertices.

(4) **(Example of projection of moment polytopes)**

(a) Give an approximate sketch of the image $\mu_H(M)$ in the case when $M = \mathbb{C}P^3$, $T = U(1)^3 \subset U(1)^4$ acting in the usual way (so the moment polytope is a 3-simplex in $\mathbb{R}^4$) and $H = U(1)^2$ acting via some embedding in $U(1)^3$: choose the orthocomplement of Lie(H) in Lie(T) to be some axis $\mathbb{R}\hat{v}$ through the origin in $\mathbb{R}^3$. (The image will depend on the axis $\mathbb{R}\hat{v}$ chosen: draw the axis you have chosen.)

(b) What are the *regular values* for the moment map of H?