

HOMEWORK SET #3: DUE OCTOBER 15

- (1) There are 6 points inside a 5×6 square. Prove that 2 of these points are at most $\sqrt{13}$ apart.
- (2) For a natural number $n \geq 2$, $n + 1$ numbers are selected from $\{1, 2, \dots, 2n\}$. Prove that two of the selected numbers, a, b , are such that a divides b .
- (3) At a party with 10 people, some pairs of people are friends. Prove that two of the people in attendance have the same number of friends at the party (*Note: these two people don't have to be friends!*)
- (4) Show that n has an odd number of divisors if and only if n is a square.
- (5) Find all prime numbers p such that $17p + 1$ is a square.
- (6) Let $a, n \geq 2$ be positive integers. Prove that if $a^n - 1$ is a prime number then $a = 2$ and n is prime.