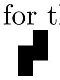
HOMEWORK SET #2: DUE SEPTEMBER 30

(1) A 9×9 grid gets covered by 16 5×1 tiles and a single 1×1 tile. What are the possible positions for the 1×1 tile?

(2) Consider the usual tetris piece:  Prove that one cannot tile an 20×20 board with 100 such pieces (rotations and reflections are allowed).

(3) Consider the tetris piece above. Suppose a 9×9 board is tiled with such pieces together with 1×1 pieces. What is the most number of tetris pieces that can be used?

(4) Prove that -shaped L-tetrominos (4 squares, width 2, height 3) cannot tile a 10×10 board.

(5) An infinite chessboard has a positive integer written in every square. The value in each square is the avaerage of the values in the four squares around it. Prove that all the numbers in all the squares are equal.

(6) Is it possible to find 100 consecutive positive integers with exactly 7 primes among them?

(7) On a large flat field, 235 people are positioned so that for each person the distances to all the other people are different. Everyone has a water pistol and at a given time fires the pistol on the person nearest to them. Show that at least 1 person is left dry.

(8) n red points and n blue points are drawn in the plane such that no 3 points lie on a line. Prove we can join each red poit to a single blue point by a line segment, such that no 2 line segments cross.

(9) In a school there are n kids, such that every kid is friends with exactly 3 others (friendship is mutual, so if A is friends with B then B is friends with A). Prove that we can split the kids up into 2 rooms, such that every kid is friends with at most 2 others in the same room.