HOMEWORK SET #1: DUE SEPTEMBER 19

(1) There are 13 cards in a jar, 8 red and 5 black. Johnny plays the following game. He removes 2 cards at a time, and then does the following:
- If the cards are both black, he puts 1 black card back.
- If the cards are one of each color, he puts 1 red card back.
- If the cards are both red, he puts an additional black card in the jar.

At the end there is a single card remaining in the jar. Is it red or black? Prove you answer. *Note: you must prove that no matter how Johnnys game turns out, the last card will be of the same colour. It is not enough to simply check one case.*

(2) There are 9 green, 12 blue, and 17 red chameleons. Every time two chameleons of different colours meet, they both switch to the third colour. Can the chameleons all become red? What about blue? What about green?

(3) Prove a 66×7 board **can** be tiled by 2×3 rectangles.

(4) There are 8 bankers standing in a row, from left to right. Initially, the right-most banker has 100 gold coins and the other bankers have non. Any given banker is allowed the following operation: she may throw away 1 gold coin, and pass another gold coin to the banker on her left. So, for example, after 1 turn the right-most banker could have 98 cold coins and the banker on her left would have 1 gold coin. Is it possible for the leftmost banker to eventually get a gold coin?

(5) The numbers 1,2,3,…,100 are written on the blackboard. You may choose 2 numbers a, b and replace them with $ab + a + b$. Eventually one number is left. What are the possibilities for this final number?

(6) **Bonus Question:** A deck contains cards numbers $1, \ldots, n$ in an arbitrary order. Repeat the following procedure: Whenever the card at the top of the deck is numbered k, reverse the order of the first k cards. Prove that the card numbered 1 eventually appears at the top of the deck. *Hint: Suppose the top of the deck is k and you reverse the order. What has to happen for k to appear at the top of the deck again?*