

Levi decompositions

Let \mathfrak{g} be a Lie algebra. The radical of \mathfrak{g} is the maximal solvable ideal of \mathfrak{g} . The Levi decomposition of \mathfrak{g} expresses \mathfrak{g} as the direct sum of its radical and a semisimple subalgebra of \mathfrak{g} . (See, for example, Appendix B of Knapp (Lie groups, beyond an introduction), or §3.14 of Varadarajan. There are many references for this.)

Now suppose that G is a Lie group. The radical of G is the largest solvable connected Lie subgroup of G . The Levi decomposition of G expresses G as a certain product of a semisimple Lie subgroup and the radical of G . (See, for example, Theorem 3.18.13 of Varadarajan's book.)

One possible approach is to learn how the theorems re Levi decompositions are proved, and to look at some examples of Levi decompositions. (*Remark:* There are some decompositions of parabolic subalgebras and parabolic subgroups that are also referred to as Levi decompositions. They are not the same as the ones mentioned above, although there is a close relationship.)