Homework Assignment 20

Assigned Tuesday March 8; not to be submitted.

Required reading. All of Spivak's Chapter 20.

In class review problem(s) (to be solved in class on Tuesday March 15). Chapter 20 problem 16:

1. Prove that if $f''(a)$ exists, then

$$f''(a) = \lim_{h\to 0} \frac{f(a+h)+f(a-h)-2f(a)}{h^2}.$$

   The limit on the right is called the *Schwarz second derivative* of f at a. Hint: Use the Taylor polynomial $P_{2,a}(x)$ with $x=a+h$ and with $x=a-h$.

2. Let $f(x)=x^2$ for $x\geq 0$ and $-x^2$ for $x\leq 0$. Show that

$$\lim_{h\to 0} \frac{f(0+h)+f(0-h)-2f(0)}{h^2}$$

   exists, even though $f''(0)$ does not.

3. Prove that if f has a local maximum at a, and the Schwartz second derivative of f at a exists, then it is ≤ 0.

4. Prove that if $f'''(a)$ exists, then

$$\frac{f'''(a)}{3} = \lim_{h\to 0} \frac{f(a+h)-f(a-h)-2hf'(a)}{h^3}.$$

Recommended for extra practice. From Spivak's Chapter 20: Problems 3, 4, 5, 6, 9, 18 and 20.

Just for fun. According to your trustworthy professor, $P_{2n+1,0,\sin}(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ should approach $\sin x$ when n goes to infinity. Here are the first few values of $P_{2n+1,0,\sin}(157)$:

n	$P_{2n+1,0,\sin}(157)$
0	157.0
1	-644825.1666
2	794263446.1416
3	-465722259874.7894
4	159244913619814.5429
5	-35629004757275297.7787
6	5619143855101017161.3172
7	-658116552443218272478.0047
8	59490490719826164706638.3418
9	-4275606060900548165855463.4918
10	250142953226934230105633222.4574
100	$\sim 5.653 \cdot 10^{63}$

In widths of hydrogen atoms that last value is way more than the diameter of the observable universe. Yet surely you remember that $|\sin 157| \leq 1$; in fact, my computer tells me that $\sin 157$ is approximately -0.0795485. In the light of that and in the light of the above table, do you still trust your professor?

The Small Print. For $n = (200,\ 205,\ 210,\ 215,\ 220)$ we get

$P_{2n+1,0,\sin}(157) = (1.8512 \cdot 10^8,\ -13102.9,\ 0.648331,\ -0.0795805,\ -0.0795485)$.