Dror Bar-Natan: Classes: 2004-05: Math 157 - Analysis I:

Homework Assignment 2

Assigned Tuesday September 21; due Friday October 1, 2PM, at SS 1071

Required reading. Like last week, read, reread and rereread your notes from this week's classes, and make sure that you really, really really, really really really understand everything in them. Do the same every week! Also, read all of Spivak's Chapter 1.

To be handed in. From Spivak Chapter 1: 11 odd parts, 12 odd parts, 14, and also

1. Show that if $a > 0$, then $ax^2 + bx + c \geq 0$ for all values of x if and only if $b^2 - 4ac \leq 0$.
2. Prove the Cauchy-Schwartz inequality

$$\left(a_1b_1 + a_2b_2 + \cdots a_nb_n\right)^2 \leq \left(a_1^2 + \cdots + a_n^2\right)\left(b_1^2 + \cdots + b_n^2\right)$$

   in two different ways:

   (a) Use $2xy \leq x^2 + y^2$ (why is this true?), with

$$x = \frac{|a_i|}{\sqrt{a_1^2 + \cdots + a_n^2}} \qquad y = \frac{|b_i|}{\sqrt{b_1^2 + \cdots + b_n^2}}$$

   (b) Consider the expression

$$(a_1x + b_1)^2 + (a_2x + b_2)^2 + \cdots + (a_nx + b_n)^2,$$

   collect terms, and apply the result of Problem 1.

Recommended for extra practice. From Spivak Chapter 1: 7, 15, 18, 20, 21, 22, 23.

Just for fun. Can you draw 3 linked circles, with the property that if any one of them disappears, the other two are no longer linked? Can you draw 4 linked circles, with the property that if any one of them disappears, the other 3 are no longer linked? What about more than 4? (In the example below, if you drop any of the components the other two remain linked).