

Math 157 Analysis I — Term Exam 2

University of Toronto, December 1, 2003

Name: _____ Student ID: _____

Solve the following 4 problems. Each is worth 25 points although they may have unequal difficulty. Write your answers in the space below the problems and on the front sides of the extra pages; use the back of the pages for scratch paper. Only work appearing on the front side of pages will be graded. Write your name and student number on each page. If you need more paper please ask the tutors. You have an hour and 45 minutes.

Allowed Material: Any calculating device that is not capable of displaying text.

Good Luck!

For Grading Use Only

1	/25
2	/25
3	/25
4	/25
Total	/100

Web version: <http://www.math.toronto.edu/~drorbn/classes/0304/157AnalysisI/TE2/Exam.html>

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Problem 1. Let $f(x)$ and $g(x)$ be continuous functions defined for all x , and assume that $f(0) = g(0)$. Define a new function $h(x)$ by

$$h(x) = \begin{cases} f(x) & x \leq 0, \\ g(x) & x \geq 0. \end{cases}$$

Is $h(x)$ continuous for all x ? Prove or give a counterexample.

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Problem 2. We say that a function f is *locally bounded* on some interval I if for every $x \in I$ there is an $\epsilon > 0$ so that f is bounded on $I \cap (x - \epsilon, x + \epsilon)$. Prove that if a function f (continuous or not) is locally bounded on a closed interval $I = [a, b]$ then it is bounded (in the ordinary sense) on that interval.

Hint. Consider the set $A = \{x \in I : f \text{ is bounded on } [a, x]\}$ and think about P13.

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Problem 3.

1. Prove that if a function g satisfies $g' \equiv 0$ on \mathbb{R} then $g \equiv c$ for some constant c .
2. A certain function f was differentiated twice, and to everybody's surprise, the result was back the function f again, except with the sign reversed: $f'' = -f$. It was also found that $f(0) = f'(0) = 1$. Set $g(x) = (f(x))^2 + (f'(x))^2$ and compute $g'(x)$, $g(0)$ and $g(157)$ (making sure that you explain every step of your computation).

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Problem 4. Draw a detailed graph of the function

$$f(x) = 4\frac{x-1}{x^2}.$$

Your drawing must clearly indicate the domain of definition of f , all intersections of the graph of f with the axes, the behaviour of f far out near $\pm\infty$ and near the boundaries of its domain of definition, the regions on which it is increasing or decreasing, the regions on which it is convex or concave and all local and global minima and maxima of f .

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