Dror Bar-Natan: Classes: 2003-04: Math 157 - Analysis I:

## Homework Assignment 21

Assigned Tuesday March 2; due Friday March 12, 2PM, at SS 1071

Required reading. Spivak's Chapter 20.
To be handed in. From Spivak Chapter 20: Odd parts of 1, 3, 4, 8 .
Recommended for extra practice. From Spivak Chapter 20: Even parts of 1, 3, 4, 8 and all of 6,9 .
Just for fun. According to your trustworthy professor, $P_{2 n+1,0, \sin }(x)=\sum_{k=0}^{n}(-1)^{k} \frac{x^{2 k+1}}{(2 k+1)!}$ should approach $\sin x$ when $n$ goes to infinity. Here are the first few values of $P_{2 n+1,0, \sin }(157)$ :

| $n$ | $P_{2 n+1,0, \text { sin }}(157)$ |
| :---: | :---: |
| 0 | 157.0 |
| 1 | -644825.1666 |
| 2 | 794263446.1416 |
| 3 | -465722259874.7894 |
| 4 | 159244913619814.5429 |
| 5 | -35629004757275297.7787 |
| 6 | 5619143855101017161.3172 |
| 7 | -658116552443218272478.0047 |
| 8 | 59490490719826164706638.3418 |
| 9 | -4275606060900548165855463.4918 |
| 10 | 250142953226934230105633222.4574 |
| 100 | $\sim 5.653 \cdot 10^{63}$ |

In widths of hydrogen atoms that last value is way more than the diameter of the observable universe. Yet surely you remember that $|\sin 157| \leq 1$; in fact, my computer tells me that $\sin 157$ is approximately -0.0795485 . In the light of that and in the light of the above table, do you still trust your professor?

