Dror Bar-Natan: Classes: 2003-04: Math 157 - Analysis I:

Homework Assignment 21

Assigned Tuesday March 2; due Friday March 12, 2PM, at SS 1071

Required reading. Spivak's Chapter 20.

To be handed in. From Spivak Chapter 20: Odd parts of 1, 3, 4, 8.

Recommended for extra practice. From Spivak Chapter 20: Even parts of 1, 3, 4, 8 and all of 6, 9.

Just for fun. According to your trustworthy professor, $P_{2n+1,0,\sin}(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ should approach $\sin x$ when n goes to infinity. Here are the first few values of $P_{2n+1,0,\sin}(157)$:

n	$P_{2n+1,0,\sin}(157)$
0	157.0
1	-644825.1666
2	794263446.1416
3	-465722259874.7894
4	159244913619814.5429
5	-35629004757275297.7787
6	5619143855101017161.3172
7	-658116552443218272478.0047
8	59490490719826164706638.3418
9	-4275606060900548165855463.4918
10	250142953226934230105633222.4574
100	$\sim 5.653 \cdot 10^{63}$

In widths of hydrogen atoms that last value is way more than the diameter of the observable universe. Yet surely you remember that $|\sin 157| \le 1$; in fact, my computer tells me that $\sin 157$ is approximately -0.0795485. In the light of that and in the light of the above table, do you still trust your professor?

 $\textbf{The Small Print.} \quad \text{For } n = (200, \ 205, \ 210, \ 215, \ 220) \text{ we get } P_{2n+1,0,\sin}(157) = (1.8512 \cdot 10^8, \ -13102.9, \ 0.648331, \ -0.0795805, \ -0.0795485).$