ALGEBRAIC TOPOLOGY, PROBLEM SET 9

1. Exact sequences and diagram chasing

Exercise 1.1. Given a triple $B \subset A \subset X$, let $k\colon (A, \emptyset) \to (A, B)$ is an inclusion. Define $\partial_q(X, A, B)\colon H_q(X, A) \to H_{q-1}(A, B)$ to be the composition

$$\partial_q(X, A, B) = H_{q-1}(k) \circ \partial_{q-1}(X, A)\colon H_q(X, A) \to H_{q-1}(A) \to H_{q-1}(A, B)$$

Prove that inclusions $i: (A, B) \hookrightarrow (X, B)$ and $j: (X, B) \hookrightarrow (X, A)$ fit into the long exact sequence

$$\ldots \to H_q(A, B) \xrightarrow{H_q(i)} H_q(X, B) \xrightarrow{H_q(j)} H_q(X, A) \xrightarrow{\partial_q(X,A,B)} H_{q-1}(A, B) \to \ldots$$

2. Calculations of singular homology

Exercise 2.1. Consider a pair of spaces (X, A), s.t. A is a retract of X. Then $H_*(X) \cong H_*(A) \oplus H_*(X, A)$

Exercise 2.2. Conclude from the previous exercise that S^n is not a retract of D^{n+1} and prove the n-dimensional Brouwer theorem.

Solve exercise 16, 17, 20 on page 132 in the Hatcher's book.

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