ALGEBRAIC TOPOLOGY, PROBLEM SET 5

1. Covering maps: Action of the fundamental group of the base on the fiber

Exercise 1.1. Let $(\tilde{X}, p)$ be a covering space of X. Then the group of automorphisms of the covering space $A(\tilde{X}, p)$ is isomorphic to the group of automorphisms of a set $p^{-1}(x), x \in X$ considered as a $\pi_1(X, x)$-space.

Exercise 1.2 (Continued)**.** For any two points $x \in X$ and $\tilde{x} \in p^{-1}(x)$

$$A(\tilde{X}, p) \cong N(p_* \pi_1(\tilde{X}, \tilde{x}))/p_* \pi_1(\tilde{X}, \tilde{x}),$$

where $N(\,\cdot\,)$ is the normalizer in $\pi_1(X, x)$.

What happens if $\pi_1(\tilde{X}, \tilde{x})$ is normal? Discuss the notion of a normal covering space. What can you say about the case of the universal covering space?

2. Applications

Solve Exercises 15, 17 from page 80.

Exercise 2.1. Find the fundamental group of a real projective space $\mathbb{R}P^n, n \geq 2$.

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