ALGEBRAIC TOPOLOGY, PROBLEM SET 4

1. Covering maps. Applications of the lifting theorems.

Solve Exercises 8, 9 from page 79. Use various lifting properties you've recently learnt. The hint for the Exercise 8 in the Hatcher's book seems misleading. Note, that a map is called nullhomotopic if it is homotopic to a map into a point. Or, equivalently, a map is nullhomotopic if it factors through a contractible space (Why ?).

2. More properties of covering maps.

Exercise 2.1. Let G be a connected and locally linearly connected topological group (a topological space G with a distinguished point $e \in G$ and two continuous maps: multiplication $\mu : G \times G \to G$ and inverse $\nu : G \to G$ satisfying obvious axioms). And let $(\tilde{G}, p)$ be a covering space. Show that $\tilde{G}$ may be given a structure of topological group.

Exercise 2.2. Let G be a group which acts freely and discretely (this means that any point has a neighbourhood U such that all open sets $gU, g \in G$ are pairwise disjoint) on a space X. Then the natural projection $X \to X/G$ is a covering map.

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