Dror's Dream Map of Quantum Groups $\subset$ Knot Theory

Drinfel'd, Etingof–Kazhdan ⊂ Prehistoric

Hallucinations

about Khovanov



C A T E G O R I F I C A T I O N



Some harsh reality.

EK mix tangles and braids and algebras and Verma modules

Night Time Dreams

about Etingof – Kazhdan

$\mathcal{K}$ / translucent vertices → $\hat{\mathcal{K}}/tv$ → $\mathrm{gr}\,\hat{\mathcal{K}}/tv$ || $\mathcal{A}/tv$ (with Z/tv)

Day Time Dreams

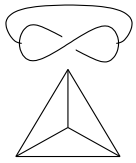
about Reshetikhin – Turaev & quasi triangular quasi Hopf algebras

Gens and rels? → $\mathcal{K}_V$ = K(T)VG, Knotted (trivalent?) virtual knots → $\hat{\mathcal{K}}_V$ → $\mathrm{gr}\,\hat{\mathcal{K}}_V$ || $\vec{\mathcal{A}}$ —W→ specific quantum groups

Reality

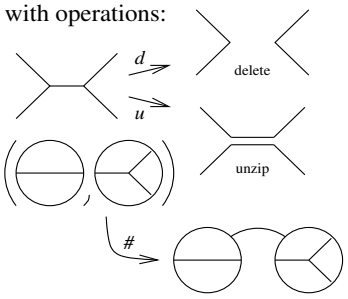
of associators, FT invariants and co–commutative quasi–Hopf algebras

Generators → $\mathcal{K}$ = KTG, Knotted Trivalent Graphs with operations:



Relations

A 5 term relation, a 6 term relation, and lesser ones.



Definable subsets:

Genus g knots, ribbon knots, boundary links and more...

$\hat{\mathcal{K}}$ —GT→ The unipotent completion —Z→ $\mathrm{gr}\,\hat{\mathcal{K}}$ // $\mathcal{A}$ —GRT→ $W_{sl(2)}$, $W_{sl(n)}$ → $U(sl(2))$, $U(sl(n))$, etc.

This is the Drinfel'd theory of Associators!

Why should we care?

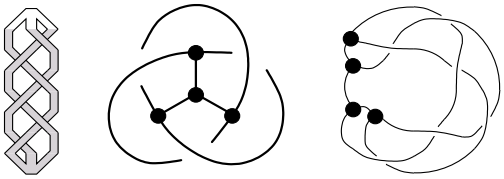
1. Usefulness!
2. Beauty!
3. Guidance!
4. Confidence!
5. Grothendieck–Teichmuller!



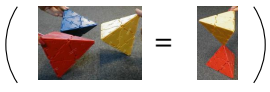
Fushimi–Inari

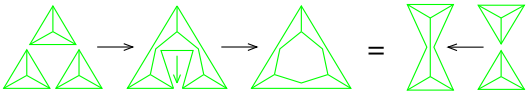
A La Carte Drawings

Knotted Trivalent Graphs (KTG's):



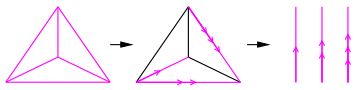
The 5–term relation:



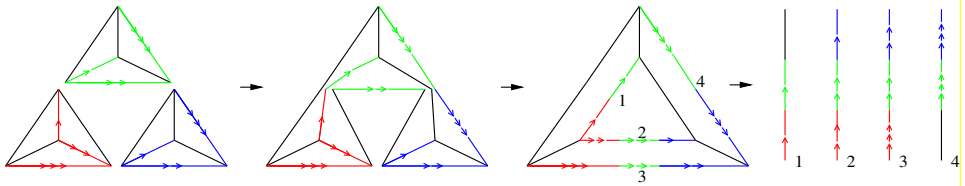


Claim. With $\Phi := Z(\triangle)$, the above relation is equivalent to the Drinfel'd's pentagon equation.

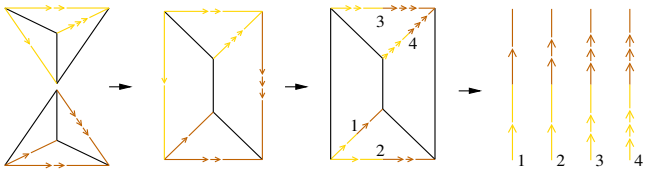
Proof.



$:= \Phi \in \mathcal{A}(\uparrow_3)$



$= (\Phi \otimes 1) \cdot (1 \otimes \Delta \otimes 1)(\Phi) \cdot (1 \otimes \Phi) \in \mathcal{A}(\uparrow_4)$



$= (\Delta \otimes 1 \otimes 1)(\Phi) \cdot (1 \otimes 1 \otimes \Delta)(\Phi) \in \mathcal{A}(\uparrow_4)$


Drinfel'd


Etingof


Kazhdan


J. Murakami


Ohtsuki


Kauffman


Goussarov


Polyak


Viro


D. Thurston


Haviv

More at http://www.math.toronto.edu/~drorbn/Talks/Kyoto-0705/