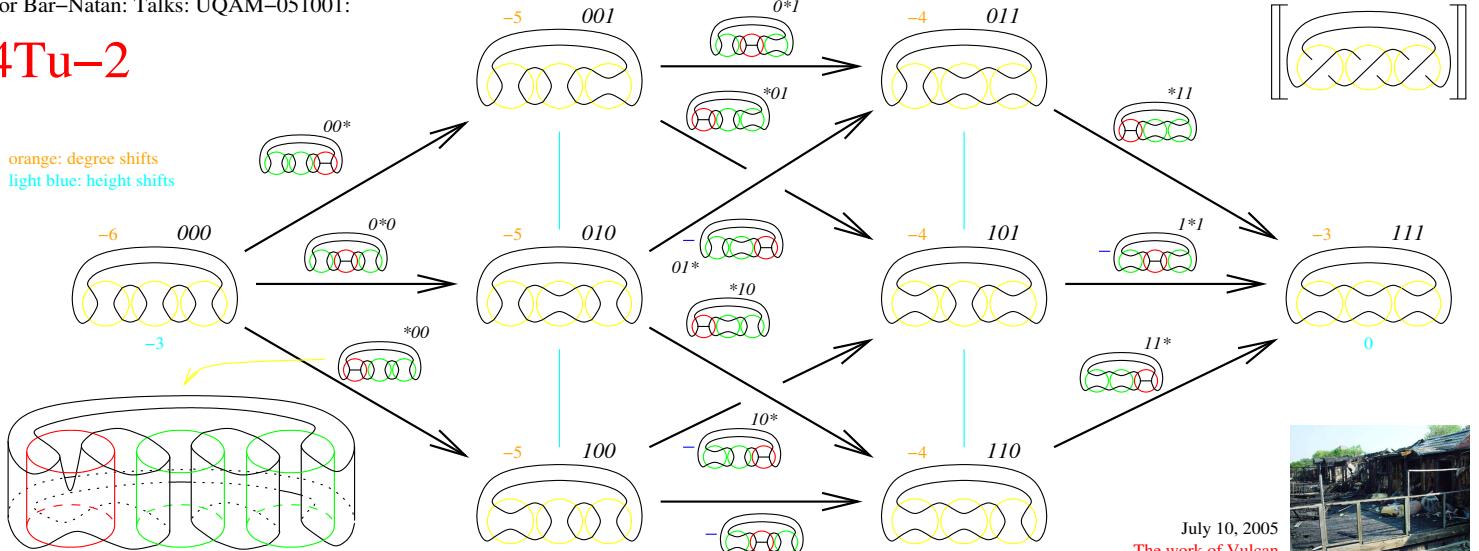


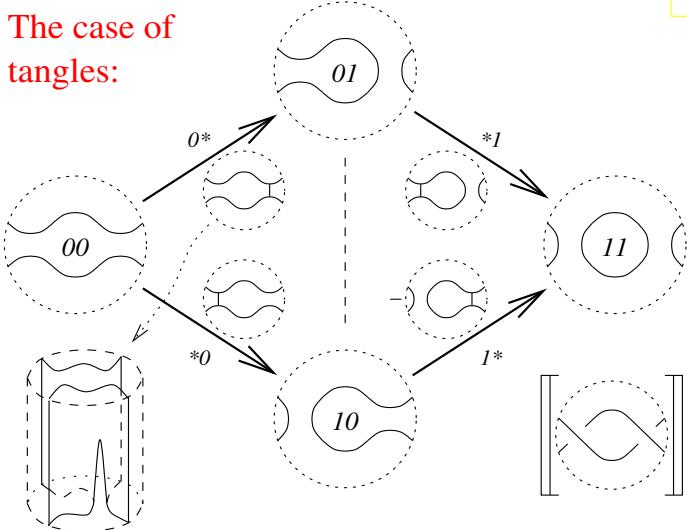
4Tu-2



General Crossings

$$\begin{array}{c} \text{Diagram 1} \rightarrow \left(\begin{array}{c} \text{Diagram 2} \\ 0 \\ +1 \end{array} \right) \xrightarrow{\text{Diagram 3}} \left(\begin{array}{c} \text{Diagram 4} \\ +1 \\ +2 \end{array} \right) \\ \text{Diagram 5} \rightarrow \left(\begin{array}{c} \text{Diagram 6} \\ -1 \\ -2 \end{array} \right) \xrightarrow{\text{Diagram 7}} \left(\begin{array}{c} \text{Diagram 8} \\ 0 \\ -1 \end{array} \right) \end{array}$$

The case of tangles:



The work of Naot.

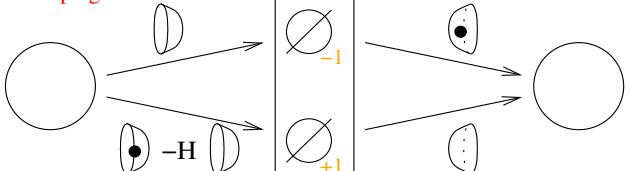
<surfaces>/4Tu is freely generated by Shrek surfaces



A Shrek surface with 7 boundaries (one distinguished), 3 handles and 2 tubes

Let \bullet denote a tube to the distinguished component (the curtain), and let H denote a handle on the curtain. Then

Delooping:

... so the invariant is valued in complexes over a category with just one object and morphisms in $Z[H]$; all is graded and $\deg H = -2$.

Where does it live? In *Kom(Mat(<Cob> / {S, T, 4Tu}) / homotopy*

Kom: Complexes *Mat*: Matrices *Cob*: Cobordisms $<\dots>$: Formal lin. comb.

$$\begin{array}{l} S: \text{Diagram} = 0 \\ T: \text{Diagram} = 2 \end{array}$$

$$\text{Diagram} + \text{Diagram} \stackrel{4Tu}{=} \text{Diagram} + \text{Diagram}$$

Invariant!



$$\begin{array}{c} \text{Diagram} \xrightarrow{F} \text{Diagram} \xrightarrow{G} \text{Diagram} \\ \text{Diagram} \xrightarrow{F_0} \text{Diagram} \xrightarrow{G_0} \text{Diagram} \\ h \text{ (horizontal)} \quad v \text{ (vertical)} \\ FG - I + h \delta = \text{Diagram} - \text{Diagram} + \text{Diagram} \end{array}$$

The Reduction Lemma. If ϕ is an isomorphism then the complex

$$[C] \xrightarrow{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & \delta \\ \gamma & \epsilon \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} \mu & \nu \end{pmatrix}} [F]$$

is isomorphic to the (direct sum) complex

$$[C] \xrightarrow{\begin{pmatrix} 0 \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & 0 \\ 0 & \epsilon - \gamma\phi^{-1}\delta \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} 0 & \nu \end{pmatrix}} [F]$$

The work of Green.



The universal invariant of the left-handed trefoil is

$$\begin{array}{c} \text{Diagram} \xrightarrow{H} \text{Diagram} \xrightarrow{0} \text{Diagram} \xrightarrow{-2} \text{Diagram} \\ -3 \quad -8 \quad -2 \quad -6 \quad -1 \quad 0 \quad -2 \end{array}$$

standard data: $\begin{matrix} -1 \\ -3 \\ -5 \\ -7 \\ -9 \\ -3 & -2 & -1 & 0 \end{matrix}$

(and the invariant of the 48 crossing T(8,7) is computable in minutes...)

Some functors.

classical	reduced	Lee	?
$H \mapsto \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array}$	$\begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array} \xrightarrow{0} \begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array}$	$\begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array}$	$\frac{\mathbb{Z}[X]}{X^2 - hX - t}$
$\begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array}$	$\begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array} \xrightarrow{0} \begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array}$	$\begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array}$	$\begin{array}{c} \langle 2 \rangle \\ \langle 0 \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle 2 \rangle \\ \langle 0 \rangle \end{array}$
$\begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array}$	$\begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array} \xrightarrow{0} \begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array}$	$\begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array}$	$\begin{array}{c} \langle 2 \rangle \\ \langle 0 \rangle \end{array} \xrightarrow{2} \begin{array}{c} \langle 2 \rangle \\ \langle 0 \rangle \end{array}$

(Lee's spectral sequence and Rasmussen's invariant also recoverable)

<http://www.math.toronto.edu/~drorbn/Talks/UQAM-051001/>