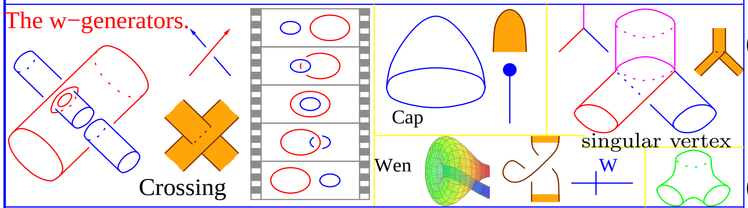
2. w−Knots, Alekseev−Torossian, and baby Etingof−Kazhdan

Dror Bar−Natan, Montpellier, June 2010, http://www.math.toronto.edu/~drorbn/Talks/Montpellier−1006/

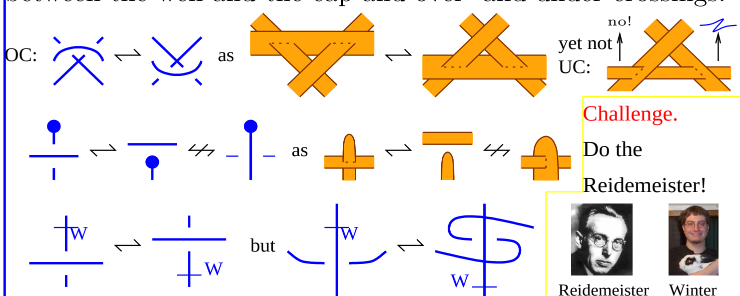
Trivalent w-Tangles.

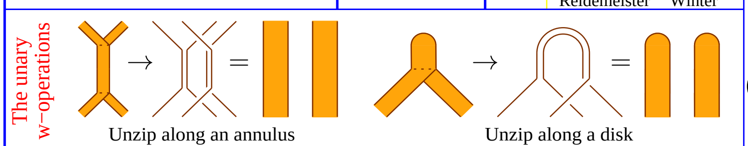
$$\text{wTT} = \text{CA}\left\langle \begin{matrix}\text{w-}\\ \text{generators}\end{matrix} \middle| \begin{matrix}\text{w-}\\ \text{relations}\end{matrix} \middle| \begin{matrix}\text{unary w-}\\ \text{operations}\end{matrix} \right\rangle$$

The w−generators.

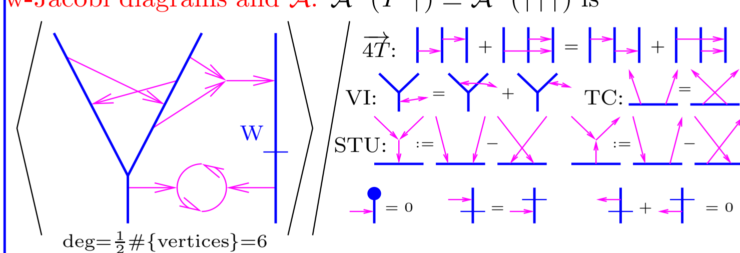


The w-relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC, $W^2 = 1$, and funny interactions between the wen and the cap and over- and under-crossings:





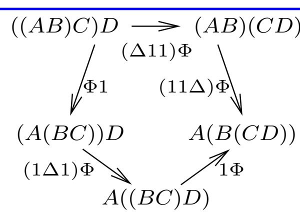
w-Jacobi diagrams and $\mathcal{A}$. $\mathcal{A}^w(Y\uparrow) \cong \mathcal{A}^w(\uparrow\uparrow\uparrow)$ is



An Associator:

$$(AB)C \xrightarrow{\Phi\in\mathcal{U}(\mathfrak{g})^{\otimes 3}} A(BC)$$

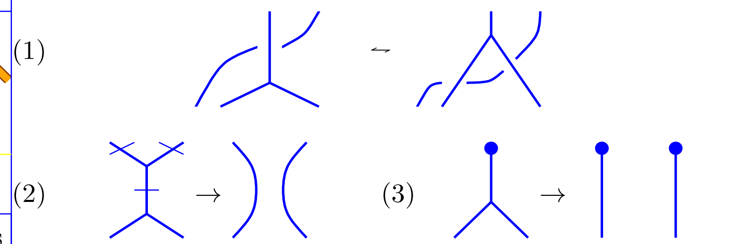
satisfying the "pentagon",



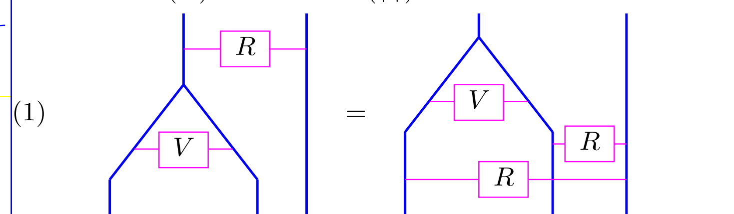
$$\Phi 1\cdot(1\Delta 1)\Phi\cdot 1\Phi = (\Delta 11)\Phi\cdot(11\Delta)\Phi$$

The hexagon? Never heard of it.

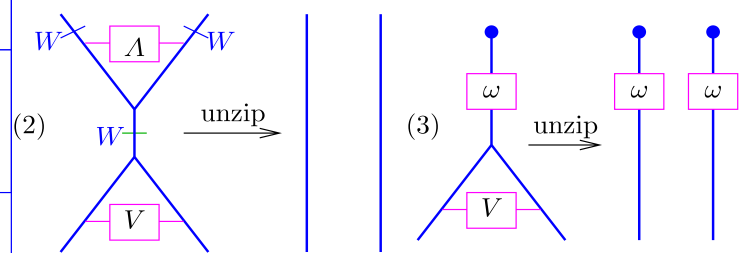
Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect $R4$ and intertwine annulus and disk unzips:



Diagrammatic statement. Let $R = \exp \text{↦} \in \mathcal{A}^w(\uparrow\uparrow)$. There exist $\omega \in \mathcal{A}^w(\uparrow)$ and $V \in \mathcal{A}^w(\uparrow\uparrow)$ so that



(1) $V\cdot(\Delta\otimes 1)(R) = R^{13}R^{23}V$ in $\mathcal{A}^w(\uparrow\uparrow\uparrow)$



(2) $VV^* = I$ in $\mathcal{A}^w(\uparrow\uparrow)$ (3) $V\cdot\Delta(\omega) = \omega\otimes\omega$ in $\mathcal{A}^w(\uparrow\uparrow)$

Alekseev-Torossian statement. There are elements $F \in \text{TAut}_2$ and $a \in \mathfrak{tr}_1$ such that

$$F(x+y) = \log e^x e^y \quad\text{and}\quad jF = a(x) + a(y) - a(\log e^x e^y).$$

Theorem. The Alekseev-Torossian statement is equivalent to the knot-theoretic statement.

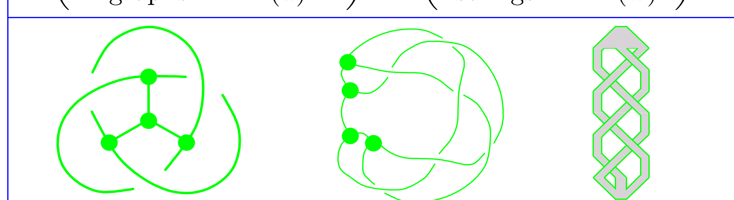
Proof. Write $V = e^c e^{uD}$ with $c \in \mathfrak{tr}_2$, $D \in \mathfrak{tder}_2$, and $\omega = e^b$ with $b \in \mathfrak{tr}_1$. Then (1) $\Leftrightarrow e^{uD}(x+y)e^{-uD} = \log e^x e^y$, (2) $\Leftrightarrow I = e^c e^{uD}(e^{uD})^* e^c = e^{2c}e^{jD}$, and (3) $\Leftrightarrow e^c e^{uD} e^{b(x+y)} = e^{b(x)+b(y)} \Leftrightarrow e^c e^{b(\log e^x e^y)} = e^{b(x)+b(y)}$ $\Leftrightarrow c = b(x) + b(y) - b(\log e^x e^y)$.

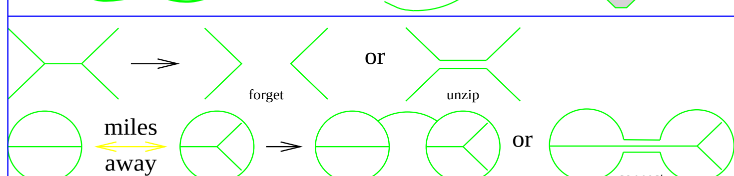
The Alekseev-Torossian Correspondence.

$$\{\text{Drinfel'd Associators}\} \leftrightarrows \{\text{Solutions of KV}\}.$$

We need an even bigger algebraic structure!

$$\left(\begin{matrix}\text{green knotted trivalent}\\ \text{graphs in } \mathbb{R}^3\ (u)\end{matrix}\right) \xrightarrow{\alpha_e} \left(\begin{matrix}\text{blue tubes and red}\\ \text{strings in } \mathbb{R}^4\ (\overline{w})\end{matrix}\right)$$





Video and more at http://www.math.toronto.edu/~drorbn/Talks/Montpellier-1006/