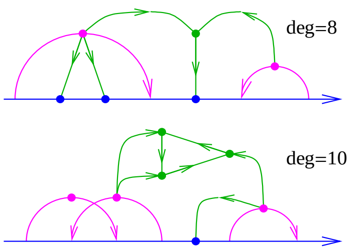
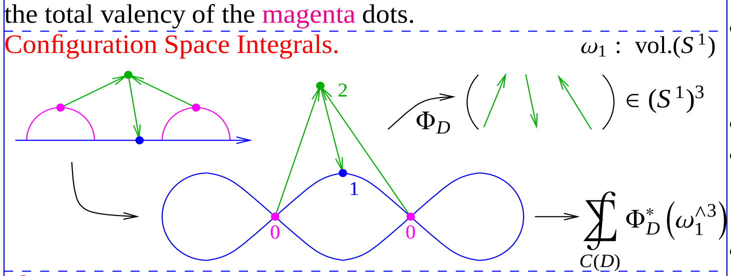
Finite Type Invariants of Doodles, 2

Feynman Diagrams and a Lower Bound on $(\mathcal{K}_0/\mathcal{K}_{n+1})^*$.

Feynman Diagrams. A blue "skeleton line" at the bottom. A magenta "arrow diagram" (directed pairing of skeleton points) on top, with a magenta dot at the middle of each arrow. A green directed graph on top, with 2-in 1-out antisymmetric green vertices, with arbitrary number of green edges starting at the magenta dots, and with some green edges terminating at distinct blue skeleton points. The *degree* is the total valency of the magenta dots.



Configuration Space Integrals.



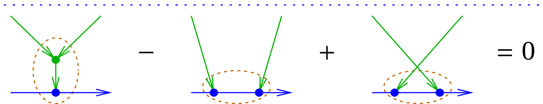
The "Partition Function" Z.

$$K \mapsto Z(K) := \sum_{\substack{\text{Feynman}\\ \text{diagrams}}} \int_{C(D)} \Phi_D^*\left(\omega_1^{\wedge e(D)}\right) \in \mathcal{A}^t := \langle D \rangle/(\partial\text{-relations}).$$

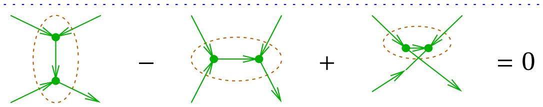
Theorem (90%). Z is an invariant of doodles.

∂-relations. STU, IHX, Foot Swap (FS), Arrow Exchange (AE), and Combinatorial R2 (CR2):

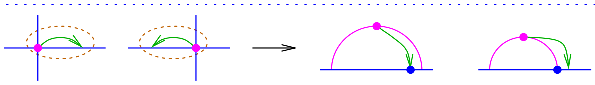
STU:



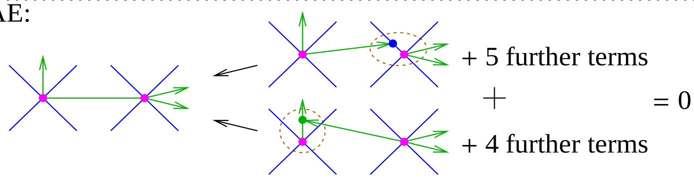
IHX:



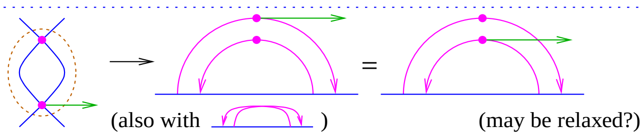
FS:



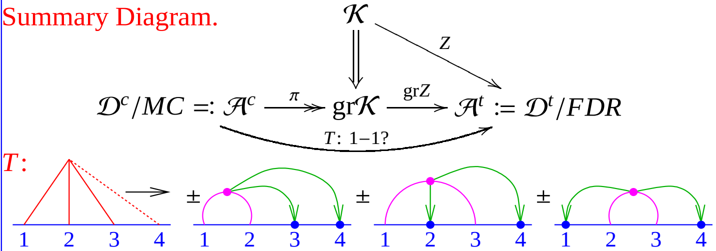
AE:



CR2:



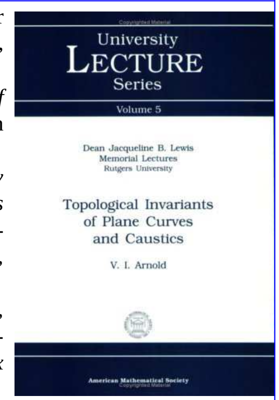
Summary Diagram.



An unfinished project!

- Nothing is written up.
- We don't know if T is injective (meaning, if our upper and lower bounds agree).
- We don't know if all of $\mathcal{A}^t$ is necessary — it is very possible that it is enough to restrict to the green-less part of $\mathcal{A}^t$ — to "Gauss Diagram Formulas".
- We haven't clarified the relationship with Merkov's [Me].
- A few further configuration space integrals can be written beyond those that we have used. We don't know what to do with those, if anything.
- We don't know the relationship, if any, with algebra.
- We don't know the relationship, if any, with quantum field theory.
- We don't know how to do similar things with 2-knots.

References. The root, of course, is [Ar]. Further references on doodles include [Kh, FT, Me, Ta, Va1, Va2]. On Goussarov finite-type: [Go, BN].



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[Kh] M. Khovanov, *Doodle Groups,* Trans. Amer. Math. Soc. **349-6** (1997) 2297–2315.

[Me] A.B. Merkov, *Vassiliev Invariants Classify Plane Curves and Doodles,* Sbornik: Mathematics **194-9** (2003) 1301.

[Ta] S. Tabachnikov, *Invariants of Smooth Triple Point Free Plane Curves,* Jour. of Knot Theory and its Ramifications **5-4** (1996) 531–552.

[Va1] V.A. Vassiliev, *On Finite Order Invariants of Triple Point Free Plane Curves,* 1999 preprint, arXiv:1407.7227.

[Va2] V.A. Vassiliev, *Invariants of Ornaments,* Adv. in Soviet Math. **21** (1994) 225–262.

open space for your doodles