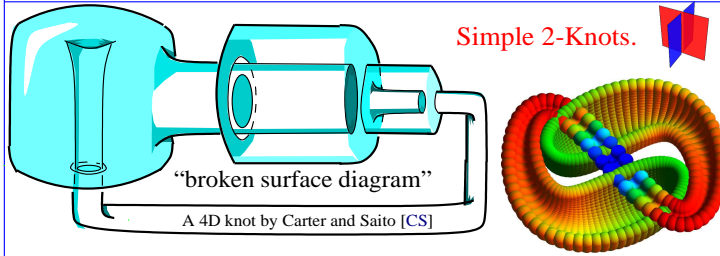
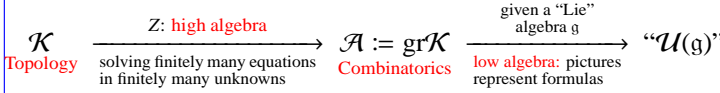


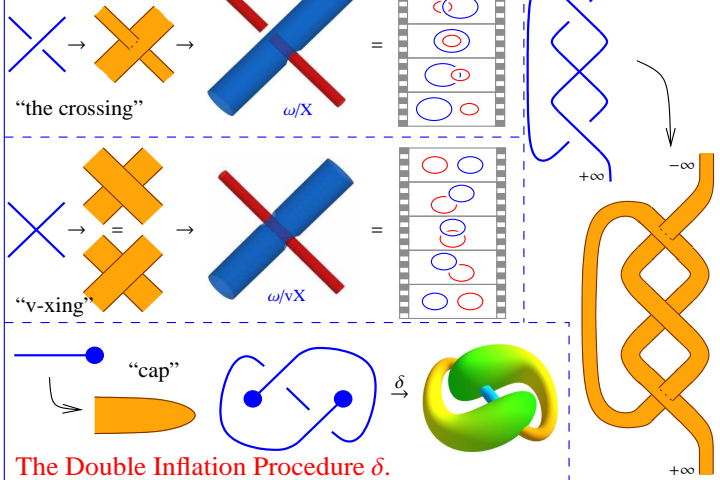


Abstract. We will repeat the 3D story of the previous 3 talks one dimension up, in 4D. Surprisingly, there's more room in 4D, and things get easier, at least when we restrict our attention to "w-knots", or to "simply-knotted 2-knots". But even then there are intricacies, and we try to go beyond simply-knotted, we are completely confused.

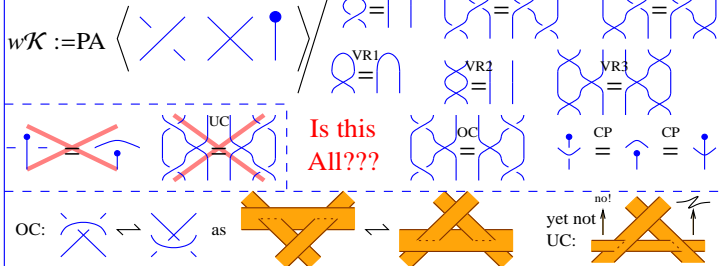
Recall.



The Generators

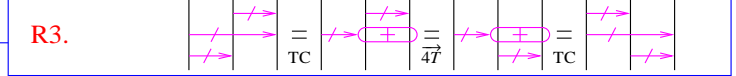
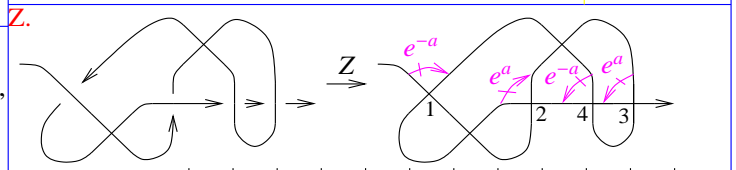
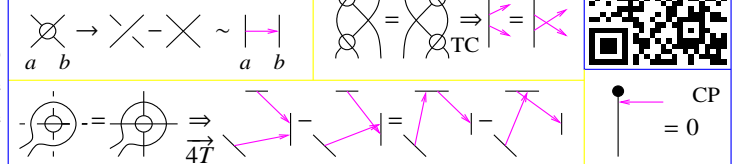


w-Knots.

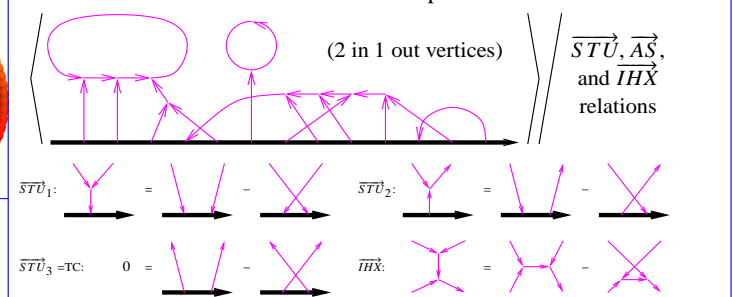


A Big Open Problem. δ maps w-knots onto simple 2-knots. To what extent is it a bijection? What other relations are required? In other words, **find a simple description of simple 2-knots.** Kawachi [Ka] may already know the answer.

The Finite Type Story.

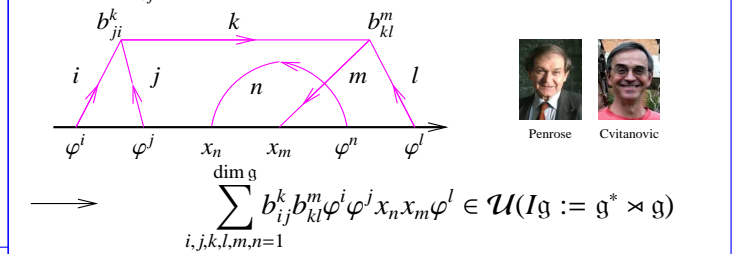


The Bracket-Rise Theorem. \mathcal{A}^w is isomorphic to



Corollaries. (1) Only wheels and isolated arrows persist:
 $\mathcal{A}^w(\uparrow_n) \cong \mathcal{U}(FL(n)_{lb}^n \ltimes CW(n))$ and $\zeta := \log Z \in FL(n)^n \times CW(n)$ has completely explicit formulas using natural FL/CW operations [BN].
 (2) Related to f.d. Lie algebras!

Low Algebra. With (x_i) and (φ^j) dual bases of \mathfrak{g} and \mathfrak{g}^* and with $[x_i, x_j] = \sum b_{ij}^k x_k$, we have $\mathcal{A}^w \rightarrow \mathcal{U}$ via



Differential Ops. We can also interpret $\hat{\mathcal{U}}(I\mathfrak{g})$ as tangential differential operators on $\text{Fun}(\mathfrak{g})$: $\varphi \in \mathfrak{g}^*$ becomes a multiplication operator, and $x \in \mathfrak{g}$ becomes a tangential derivation, in the direction of the action of $\text{ad } x$: $(x\varphi)(y) := \varphi([x, y])$.

Too easy so far! Yet once you add "foam vertices", it gets related to the Kashiwara-Vergne problem [KV] as told by Alekseev-Torossian [AT]:

