

Demo Programs for 0-Co.

ωεβ/Demo

$$R_{\theta, i, j}^+ := \mathbb{E} [b_i c_j + b_i^{-1} (e^{b_i} - 1) u_i w_j];$$

$$R_{\theta, i, j}^- := \mathbb{E} [-b_i c_j + b_i^{-1} (e^{-b_i} - 1) u_i w_j];$$

The R-matrices

CF[ω₋. E[Q₋]] := Simplify[ω E[Simplify[Q]]];
 E /: E[Q1₋] E[Q2₋] := CF@E[Q1 + Q2];
 ω1₋. E[Q1₋] ≡ ω2₋. E[Q2₋] := Simplify[ω1 == ω2 ∧ Q1 == Q2];

Utilities

N_(x:w|u)_i c_j →_k [ω₋. E[Q₋]] := CF [Normal Ordering Operators
 ω E[e^α x_k + γ c_k + (Q / . c_j | x_i → θ)] / . {γ → ∂_{c_j} Q, α → ∂_{x_i} Q};
 N_w_i u_j →_k [ω₋. E[Q₋]] := CF [
 v ω E[-b_k v α β + v β u_k + v α w_k + v δ u_k w_k + (Q / . w_i | u_j → θ)] / .
 v → (1 + b_k δ)⁻¹ / .
 {α → ∂_{w_i} Q / . u_j → θ, β → ∂_{u_j} Q / . w_i → θ, δ → ∂_{w_i, u_j} Q}];

Stitching

$$m_{i, j \rightarrow k} [Z] := \text{Module} [\{X, Z\},$$

$$\text{CF} [(Z // N_{w_i} u_{j \rightarrow x} // N_{c_i} u_{x \rightarrow x} // N_{w_x} c_{j \rightarrow x}) / . Z_{-i|j|x} \rightarrow Z_k]]$$

T₀ = R_{0,5,1}⁺ R_{0,2,4}⁺ R_{0,3,6}⁺ Some calculations for T₀

$$\mathbb{E} \left[b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{(-1+e^{b_5}) u_5 w_1}{b_5} + \frac{(-1+e^{b_2}) u_2 w_4}{b_2} + \frac{(-1+e^{-b_3}) u_3 w_6}{b_3} \right]$$

$$T_0 // m_{1,2 \rightarrow 1} // m_{3,4 \rightarrow 3} // m_{3,5 \rightarrow 3} // m_{3,6 \rightarrow 3}$$

$$\frac{1}{1 - (-1+e^{b_1}) (-1+e^{b_3})} \mathbb{E} \left[b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{e^{b_3} (-1+e^{b_1}) (-1+e^{b_3}) u_1 w_1}{(-e^{b_1} - e^{b_3} + e^{b_1+b_3}) b_1} - \frac{e^{b_1} (-1+e^{b_3}) u_3 w_1}{(-1+(-1+e^{b_1}) (-1+e^{b_3})) b_3} - \frac{e^{-b_3} (-1+e^{b_1}) u_3 w_3}{b_3} - \frac{e^{-b_3} (-1+e^{b_1}) (-e^{b_3} b_3 u_1 + e^{b_1} (-1+e^{b_3}) b_1 u_3) w_3}{b_1 (b_3 - (-1+e^{b_1}) (-1+e^{b_3}) b_3)} \right]$$

Verifying meta-associativity

$$Q_0 = \mathbb{E} [\text{Sum}[f_i c_i, \{i, 3\}] + \text{Sum}[f_{i,j} u_i w_j, \{i, 3\}, \{j, 3\}]]$$

$$\mathbb{E} [c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_1 f_{1,1} + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + u_2 w_1 f_{2,1} + u_2 w_2 f_{2,2} + u_2 w_3 f_{2,3} + u_3 w_1 f_{3,1} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}]$$

$$(Q_0 // m_{1,2 \rightarrow 1} // m_{1,3 \rightarrow 1}) \equiv (Q_0 // m_{2,3 \rightarrow 2} // m_{1,2 \rightarrow 1})$$

True

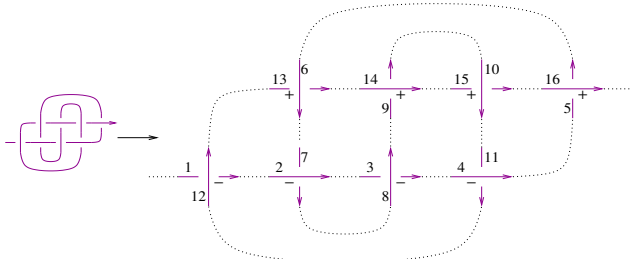
$$t_1 = R_{\theta,1,2}^+ R_{\theta,3,4}^+ R_{\theta,5,6}^+ // m_{3,5 \rightarrow x} // m_{1,6 \rightarrow y} // m_{2,4 \rightarrow z}$$

Testing R3

$$\mathbb{E} [b_x c_y + b_x c_z + b_y c_z + \frac{e^{b_x} (-1+e^{b_y}) u_y w_z}{b_y} + \frac{(-1+e^{b_x}) u_x (w_y + w_z)}{b_x}]$$

$$t_1 \equiv (R_{\theta,1,2}^+ R_{\theta,3,4}^+ R_{\theta,5,6}^+ // m_{1,3 \rightarrow x} // m_{2,5 \rightarrow y} // m_{4,6 \rightarrow z})$$

True



$$z_1 = R_{\theta,12,1}^- R_{\theta,2,7}^- R_{\theta,8,3}^- R_{\theta,4,11}^- R_{\theta,16,5}^+ R_{\theta,6,13}^- R_{\theta,14,9}^- R_{\theta,10,15}^+$$

$$\text{Do}[z_1 = (z_1 // m_{1,n \rightarrow 1}) / . b_- \rightarrow b, \{n, 2, 16\}];$$

$$\{\text{CF}@z_1, \text{KnotData}[\{8, 17\}, \text{"AlexanderPolynomial"}][t]\}$$

$$\left\{ -\frac{e^{3b} \mathbb{E}[\theta]}{1-4e^{b,8} e^{2b,11} e^{3b,8} e^{4b,4} e^{5b,6} e^b}, 11 - \frac{1}{t^3} + \frac{4}{t^2} - \frac{8}{t} - 8t + 4t^2 - t^3 \right\}$$

Demo Programs for 1-Co.

ωεβ/Demo

$$\Delta[k_-] := ((t_k - 1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 v_k c_k w_k \delta^2 \mu^2 - \delta (1 + \mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \delta^3 (1 + 3 \mu) - 2 (\alpha \beta + 2 \delta \mu + v_k w_k \delta^2 (1 + 2 \mu) + 2 c_k \delta \mu^2) (w_k \alpha + v_k \beta) - 4 (c_k \mu^2 + v_k w_k \delta (1 + \mu)) (\alpha \beta + \delta \mu) (1 + t_k) / 4; \text{The } \text{L}\acute{o}\gamma\omicron\varsigma$$

$$R_{i,j}^+ := \mathbb{E} [1, \text{Log}[t_i] c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2 / 4];$$

$$R_{i,j}^- := \mathbb{E} [1, -\text{Log}[t_i] c_j, -t_i^{-1} v_i w_j, t_i^{-1} v_i c_j w_j - c_i c_j - t_i^{-2} v_i^2 w_j^2 / 4];$$

$$(ur_{i-} := \mathbb{E} [t_i^{-1/2}, \theta, \theta, c_i t_i^2]; nr_{i-} := \mathbb{E} [t_i^{1/2}, \theta, \theta, -c_i t_i^2];)$$

The Generators

Differential Polynomials

$$DP_{x \rightarrow D_\alpha, y \rightarrow D_\beta} [P_-] [f_-] := (* \text{ means } P[\partial_\alpha, \partial_\beta] [f] *)$$

$$\text{Total}[\text{CoefficientRules}[P, \{x, y\}] / . (\{m_-, n_-\} \rightarrow c_-) \Rightarrow c D[f, \{\alpha, m\}, \{\beta, n\}]]$$

Utilities

$$\text{CF}[\mathcal{E}_- \mathbb{E}] := \text{Expand} / @ \text{Together} / @ \mathcal{E};$$

$$\mathbb{E} /: \mathbb{E}[\omega_1, L_1, Q_1, P_1] \mathbb{E}[\omega_2, L_2, Q_2, P_2] := \text{CF}@E[\omega_1 \omega_2, L_1 + L_2, \omega_2 Q_1 + \omega_1 Q_2, \omega_2^4 P_1 + \omega_1^4 P_2];$$

Normal Ordering Operators

$$N_{c_j} (x:v|w)_i \rightarrow k_- [\mathbb{E}[\omega_-, L_-, Q_-, P_-]] := \text{With}[\{q = e^{\gamma \beta} x_k + \gamma c_k\}, \text{CF}[\mathbb{E}[\omega, \gamma c_k + (L / . c_j \rightarrow \theta), \omega e^{\gamma \beta} x_k + (Q / . x_i \rightarrow \theta), e^{-q} DP_{c_j \rightarrow D_\gamma, x_i \rightarrow D_\beta} [P] [e^q]] / . \{\gamma \rightarrow \partial_{c_j} L, \beta \rightarrow \omega^{-1} \partial_{x_i} Q\}]]];$$

$$N_{w_i} v_j \rightarrow k_- [\mathbb{E}[\omega_-, L_-, Q_-, P_-]] := \text{With}[\{q = ((1 - t_k) \alpha \beta + \beta v_k + \alpha w_k + \delta v_k w_k) / \mu\}, \text{CF}[\mathbb{E}[\mu \omega, L, \mu \omega q + \mu (Q / . w_i | v_j \rightarrow \theta), \mu^4 e^{-q} DP_{w_i \rightarrow D_\alpha, v_j \rightarrow D_\beta} [P] [e^q] + \omega^4 \Delta[k]] / . \mu \rightarrow 1 + (t_k - 1) \delta / . \{\alpha \rightarrow \omega^{-1} (\partial_{w_i} Q / . v_j \rightarrow \theta), \beta \rightarrow \omega^{-1} (\partial_{v_j} Q / . w_i \rightarrow \theta), \delta \rightarrow \omega^{-1} \partial_{w_i, v_j} Q\}]]];$$

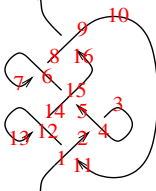
Stitching

$$m_{i, j \rightarrow k} [Z_- \mathbb{E}] := \text{Module} [\{X, Z\}, \text{CF} [(Z // N_{w_i} v_{j \rightarrow x} // N_{c_i} v_{x \rightarrow x} // N_{w_x} c_{j \rightarrow x}) / . Z_{-i|j|x} \rightarrow Z_k]]$$

$$z_2 = R_{1,11}^+ R_{4,2}^- nr_3 R_{15,5}^+ R_{6,8}^- ur_7 R_{16}^+ nr_{10} R_{12,14}^- ur_{13};$$

$$(\text{Do}[z_2 = z_2 // m_{1,k \rightarrow 1}, \{k, 2, 16\}]; z_2 = z_2 / . a_{-1} \Rightarrow a)$$

The 0-Framed Trefoil



$$\mathbb{E} [-1 + \frac{1}{t} + t, \theta, \theta, 16 + \frac{2c}{t^4} - \frac{1}{t^3} - \frac{6c}{t^3} + \frac{4}{t^2} + \frac{10c}{t^2} - \frac{10}{t} - \frac{8c}{t} - 18t + 8ct + 14t^2 - 10ct^2 - 7t^3 + 6ct^3 + 2t^4 - 2ct^4 + 2vw - \frac{2vw}{t^4} + \frac{4vw}{t^3} - \frac{6vw}{t^2} + \frac{2vw}{t} - 6tvw + 4t^2vw - 2t^3vw]$$

Questions and To Do List. • Clean up and write up. • Implement well, compute for everything in sight. • Why are our quantities polynomials rather than just rational functions? • Bounds on their degrees? • Their integrality (Z) properties? • Can everything be re-stated using integrals (∫)? • Find the 2-variable version (for knots). How complex is it? • What about links / closed components? • Fully digest the “expansion” theorem; include cuaps. • Explore the (non-)dependence on R. • Is there a canonical R? • What does “group like” mean? • Strand removal? Strand doubling? Strand reversal? • Say something about knot genus. • Find the EK/AT/KV “vertex”. • Use as a playground to study associators/braidors. • Restate in topological language. • Study the associated (v-)braid representations. • Study mirror images and the b⁺ ↔ b⁻ involution. • Study ribbon knots. • Make precise the relationship with Γ-calculus and Alexander. • Relate to the coloured Jones polynomial. • Relate with “ordinary” q-algebra. • k-smidgen sl_n, etc. • Are there “solvable” CYBE algebras not arising from semi-simple algebras? • Categorify and appease the Gods.

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