The Real Thing. In the algebra QU_ϵ, over $\mathbb{Q}[[\hbar]]$ using the *yaxt* order, $T=\mathbb{e}^{\hbar t}$, $\bar{T}=T^{-1}$, $\mathcal{A}=\mathbb{e}^{\alpha}$, and $\bar{\mathcal{A}}=\mathcal{A}^{-1}$, we have

$$\tilde{R}_{ij}=\mathbb{e}^{\hbar(y_ix_j-t_ia_j)}\left(1+\epsilon\hbar\left(a_ia_j-\hbar^2y_i^2x_j^2/4\right)+O(\epsilon^2)\right)$$

in $\mathcal{S}(B_i,B_j)$, and in $\mathcal{S}(B_1^*,B_2^*,B)$ we have

$$\tilde{m}=\mathbb{e}^{(\alpha_1+\alpha_2)a+\eta_2\xi_1(1-T)/\hbar+(\xi_1\bar{\mathcal{A}}_2+\xi_2)x+(\eta_1+\eta_2\bar{\mathcal{A}}_1)y}\left(1+\epsilon\lambda+O(\epsilon^2)\right),$$

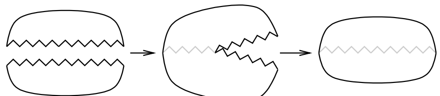
where $\lambda = 2a\eta_2\xi_1T+\eta_2^2\xi_1^2\left(3T^2-4T+1\right)/4\hbar-\eta_2\xi_1^2(3T-1)x\bar{\mathcal{A}}_2/2$
$-\eta_2^2\xi_1(3T-1)y\bar{\mathcal{A}}_1/2+\eta_2\xi_1xy\hbar\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2$.

Finally,

$$\tilde{\Delta}=\mathbb{e}^{\tau(t_1+t_1)+\eta(y_1+T_1y_2)+\alpha(a_1+a_2)+\xi(x_1+x_2)}\,(1+O(\epsilon))\in\mathcal{S}(B^*,B_1,B_2),$$

and $\tilde{S}=\mathbb{e}^{-\tau t-\alpha a-\eta\xi(1-\bar{T})\mathcal{A}/\hbar-\bar{T}\eta y\mathcal{A}-\xi x\mathcal{A}}\,(1+O(\epsilon))\in\mathcal{S}(B^*,B)$.

The Zipping Issue. (between unbound and bound lies half-zipped).



Zipping. If $P(\zeta^j,z_i)$ is a polynomial, or whenever otherwise convergent, set $\left\langle P(\zeta^j,z_i)\right\rangle_{(\zeta^j)}=P\left(\partial_{z_j},z_i\right)\Big|_{z_i=0}$. (E.g., if $P=\sum a_{nm}\zeta^nz^m$ then $\langle P\rangle_\zeta=\sum a_{nm}\,\partial_z^nz^m\big|_{z=0}=\sum n!a_{nn}$).

The Zipping / Contraction Theorem. If $P=P(\zeta^j,z_i)$ has a finite ζ-degree and the y's and the q's are "small" then

$$\left\langle P\mathbb{e}^{c+\eta^iz_i+y_j\zeta^j+q^i_jz_i\zeta^j}\right\rangle_{(\zeta^j)}=\det(\tilde{q})\mathbb{e}^{c+\eta^i\tilde{q}^k_iy_k}\left\langle P\,\middle|\,\begin{smallmatrix}\zeta^j\to\zeta^j+\eta^i\tilde{q}^j_i\\ z_i\to\tilde{q}^k_i(z_k+y_k)\end{smallmatrix}\right\rangle_{(\zeta^j)}$$

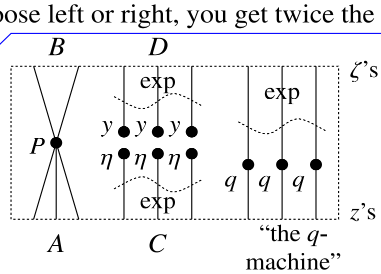
where $\tilde{q}$ is the inverse matrix of $1-q$: $(\delta^i_j-q^i_j)\tilde{q}^j_k=\delta^i_k$.

Exponential Reservoirs. The true Hilbert hotel is exp! Remove one x from an "exponential reservoir" of x's and you are left with the same exponential reservoir:

$$\mathbb{e}^x=\left[\ldots+\frac{xxxxx}{120}+\ldots\right]\xrightarrow{\partial_x}\left[\ldots+\frac{x\not{x}xxx}{120}+\ldots\right]=(\mathbb{e}^x)'=\mathbb{e}^x,$$

and if you let each element choose left or right, you get twice the same reservoir:

$$\mathbb{e}^x\xrightarrow{x\to x_l+x_r}\mathbb{e}^{x_l+x_r}=\mathbb{e}^{x_l}\mathbb{e}^{x_r}.$$



A Graphical Proof. Glue top to bottom on the right, in all possible ways. Several scenarios occur:

1. Start at A, go through the q-machine $k\geq 0$ times, stop at B. Get $\left\langle P\left(\zeta,\sum_{k\geq 0}q^kz\right)\right\rangle=\langle P(\zeta,\tilde{q}z)\rangle$.
2. Loop through the q-machine and swallow your own tail. Get $\exp\left(\sum q^k/k\right)=\exp(-\log(1-q))=\tilde{q}$. 3. …

By the reservoir splitting principle, these scenarios contribute multiplicatively. □

Implementation. ($\mathbb{E}[Q,P]$ means $\mathbb{e}^QP$) ωεβ/Zip

```
Zip_{ζs_List}@𝔼[Q_, P_] :=
  Module[{ζ, z, zs, c, ys, ηs, qt, zrule, ζrule},
   zs = Table[ζ*, {ζ, ζs}];
   c = Q /. Alternatives @@ (ζs ⋃ zs) → 0;
   ys = Table[∂_ζ (Q /. Alternatives @@ zs → 0), {ζ, ζs}];
   ηs = Table[∂_z (Q /. Alternatives @@ ζs → 0), {z, zs}];
   qt = Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ}Q, {ζ, ζs}, {z, zs}];
   zrule = Thread[zs → qt.(zs + ys)];
   ζrule = Thread[ζs → ζs + ηs.qt];
   Simplify /@
    𝔼[c + ηs.qt.ys, Det[qt] Zip_{ζs}[P /. (zrule ⋃ ζrule)]] ];
```

Real Zipping is a minor mess, and is done in two phases:

	τa-phase		ξy-phase	
ζ-like variables	τ	a	ξ	y
z-like variables	t	α	x	η

Already at $\epsilon=0$ we get the best known formulas for the Alexander polynomial!

Generic Docility. A "docile perturbed Gaussian" in the variables $(z_i)_{i\in S}$ over the ring R is an expression of the form

$$\mathbb{e}^{q^{ij}z_iz_j}P=\mathbb{e}^{q^{ij}z_iz_j}\left(\sum_{k\geq 0}\epsilon^kP_k\right),$$

where all coefficients are in R and where P is a "docile series": $\deg P_k\leq 4k$.

Our Docility. In the case of QU_ϵ, all invariants and operations are of the form $\mathbb{e}^{L+Q}P$, where

- L is a quadratic of the form $\sum l_{z\zeta}z\zeta$, where z runs over $\{t_i,\alpha_i\}_{i\in S}$ and ζ over $\{\tau_i,a_i\}_{i\in S}$, with integer coefficients $l_{z\zeta}$.
- Q is a quadratic of the form $\sum q_{z\zeta}z\zeta$, where z runs over $\{x_i,\eta_i\}_{i\in S}$ and ζ over $\{\xi_i,y_i\}_{i\in S}$, with coefficients $q_{z\zeta}$ in the ring R_S of rational functions in $\{T_i,\mathcal{A}_i\}_{i\in S}$.
- P is a docile power series in $\{y_i,a_i,x_i,\eta_i,\xi_i\}_{i\in S}$ with coefficients in R_S, and where $\deg(y_i,a_i,x_i,\eta_i,\xi_i)=(1,2,1,1,1)$.

Docililty Matters! The rank of the space of docile series to ϵ^k is polynomial in the number of variables $|S|$. !!!!!

- At $\epsilon^2=0$ we get the Rozansky-Overbay [Ro1, Ro2, Ro3, Ov] invariant, which is stronger than HOMFLY-PT polynomial and Khovanov homology taken together!
- In general, get "higher diagonals in the Melvin-Morton-Rozansky expansion of the coloured Jones polynomial" [MM, BNG], but why spoil something good?

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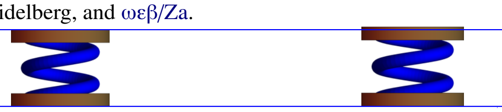

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Ohio-1901