Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we’ll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and if we’ll still have time, we’ll prove that these knots are really knotted.

Warmup: Flatlanders View an Elephant.

Knots.

Reidemeister’s Theorem. (a) Every knot has a “broken curve diagram”, made only of curves and “crossings” like \( \times \). (b) Two knot diagrams represent the same 3D knot iff they differ by a sequence of “Reidemeister moves”:

\[
R_3 = \begin{array}{c}
\begin{array}{c}
R_2 = \begin{array}{c}
R_1 =
\end{array}
\end{array}
\end{array}
\]

Topology is locally analysis and globally algebra

3-Colourings. Colour the arcs of a broken arc diagram in RGB so that every crossing is either mono-chromatic or tri-chromatic. Let \( \lambda(K) \) be the number of such 3-colourings that \( K \) has.

Example. \( \lambda(\emptyset) = 3 \) while \( \lambda(\circ) = 9; \) so \( \emptyset \neq \circ \).

Riddle. Is \( \lambda(K) \) always a power of 3?

Proof sketch. It is enough to show that for each Reidemeister move, there is an end-colours-preserving bijection between the colourings of the two sides. E.g.:

“A God created the knots, all else in topology is the work of mortals.”

Leopold Kronecker (modified)

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Cornell-150925/. Similar talks at .../CUMC-1307/,.../CUMC-1307/