The Philosophy Corner. "Universal invariants", valued in universal enveloping algebra (possibly quantized) rather than in representations thereof, are a priori better than the representation theoretic ones. They are compatible with strand doubling (the Hopf coproduct), and as the knot genus and the ribbon property for knots are expressible in terms of strand doubling, universal invariants stand a chance to say something about these properties. Indeed, they sometimes do! See e.g. [BN2, GK, LV, BG]. Representation theoretic invariants don't do that!

There's more! To get sl_2 invariants mod ϵ^3, add the following to $L(X_{ij}^+)$, $L(X_{ij}^-)$, and $L(C_i^\varphi)$, respectively (and see More.nb at ωεβ/ap for the verifications):

☺ `ϵ^2 r2[1, i, j]`

💻 $$\frac{1}{12}\epsilon^2\left(-6\,p_i\,x_i+6\,p_j\,x_i-3\,(-1+3\,T)\,p_i\,p_j\,x_i^2+3\,(-1+3\,T)\,p_j^2\,x_i^2+4\,(-1+T)\,p_i^2\,p_j\,x_i^3-2\,(-1+T)\,(5+T)\,p_i\,p_j^2\,x_i^3+2\,(-1+T)\,(3+T)\,p_j^3\,x_i^3+18\,p_i\,p_j\,x_i\,x_j-18\,p_j^2\,x_i\,x_j-6\,p_i^2\,p_j\,x_i^2\,x_j+6\,(2+T)\,p_i\,p_j^2\,x_i^2\,x_j-6\,(1+T)\,p_j^3\,x_i^2\,x_j-6\,p_i\,p_j^2\,x_i\,x_j^2+6\,p_j^3\,x_i\,x_j^2\right)$$

☺ `ϵ^2 r2[-1, i, j]`

💻 $$\frac{1}{12\,T^2}\epsilon^2\left(-6\,T^2\,p_i\,x_i+6\,T^2\,p_j\,x_i+3\,(-3+T)\,T\,p_i\,p_j\,x_i^2-3\,(-3+T)\,T\,p_j^2\,x_i^2-4\,(-1+T)\,T\,p_i^2\,p_j\,x_i^3+2\,(-1+T)\,(1+5\,T)\,p_i\,p_j^2\,x_i^3-2\,(-1+T)\,(1+3\,T)\,p_j^3\,x_i^3+18\,T^2\,p_i\,p_j\,x_i\,x_j-18\,T^2\,p_j^2\,x_i\,x_j-6\,T^2\,p_i^2\,p_j\,x_i^2\,x_j+6\,T\,(1+2\,T)\,p_i\,p_j^2\,x_i^2\,x_j-6\,T\,(1+T)\,p_j^3\,x_i^2\,x_j-6\,T^2\,p_i\,p_j^2\,x_i\,x_j^2+6\,T^2\,p_j^3\,x_i\,x_j^2\right)$$

☺ `ϵ^2 γ2[φ, i]`

💻 $$-\frac{1}{2}\epsilon^2\,\varphi^2\,p_i\,x_i$$

The sl_2 formulas mod ϵ^4 are in the last page of the handout of [BN3].

We are very close to having some sl_3 formulas, but they are certainly not ready for prime time.

References.

[BN1] D. Bar-Natan, *Everything around sl^ϵ_{2+} is DoPeGDO. So what?,* talk given in "Quantum Topology and Hyperbolic Geometry Conference", Da Nang, Vietnam, May 2019. Handout and video at ωεβ/DPG.

[BN2] D. Bar-Natan, *Algebraic Knot Theory,* talk given in Sydney, September 2019. Handout and video at ωεβ/AKT.

[BN3] D. Bar-Natan, *Cars, Interchanges, Traffic Counters, and some Pretty Darned Good Knot Invariants,* talk given in "Using Quantum Invariants to do Interesting Topology", Oaxaca, Mexico, October 2022. Handout and video at ωεβ/Cars.

[BV1] D. Bar-Natan and R. van der Veen, *A Polynomial Time Knot Polynomial,* Proc. Amer. Math. Soc. **147** (2019) 377–397, arXiv:1708.04853.

[BV2] D. Bar-Natan and R. van der Veen, *A Perturbed-Alexander Invariant,* to appear in Quantum Topology, ωεβ/APAI.

[BV3] D. Bar-Natan and R. van der Veen, *Perturbed Gaussian Generating Functions for Universal Knot Invariants,* arXiv:2109.02057.

[BG] J. Becerra Garrido, *Universal Quantum Knot Invariants,* Ph.D. thesis, University of Groningen, ωεβ/BG.

[GK] S. Garoufalidis and R. Kashaev, *Multivariable Knot Polynomials from Braided Hopf Algebras with Automorphisms,* arXiv:2311.11528.

[La] R. J. Lawrence, *Universal Link Invariants using Quantum Groups,* Proc. XVII Int. Conf. on Diff. Geom. Methods in Theor. Phys., Chester, England, August 1988. World Scientific (1989) 55–63.

[LV] D. López Neumann and R. van der Veen, *Genus Bounds from Unrolled Quantum Groups at Roots of Unity,* arXiv:2312.02070.

[Oh] T. Ohtsuki, *Quantum Invariants,* Series on Knots and Everything **29**, World Scientific 2002.

[Ov] A. Overbay, *Perturbative Expansion of the Colored Jones Polynomial,* Ph.D. thesis, University of North Carolina, August 2013, ωεβ/Ov.

[R1] L. Rozansky, *A Contribution of the Trivial Flat Connection to the Jones Polynomial and Witten's Invariant of 3D Manifolds, I,* Comm. Math. Phys. **175-2** (1996) 275–296, arXiv:hep-th/9401061.

[R2] L. Rozansky, *The Universal R-Matrix, Burau Representation and the Melvin-Morton Expansion of the Colored Jones Polynomial,* Adv. Math. **134-1** (1998) 1–31, arXiv:q-alg/9604005.

[R3] L. Rozansky, *A Universal U(1)-RCC Invariant of Links and Rationality Conjecture,* arXiv:math/0201139.

Acknowledgement. This work was supported by NSERC grant RGPIN-2018-04350 and by the Chu Family Foundation (NYC).

Disclaimer. It's fun, but not fully ready.

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Groningen-240530.