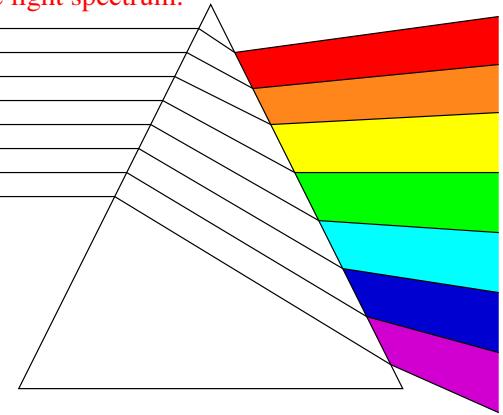


Khovanov Homology for Tangles and Cobordisms

some extra formulas and pictures

The light spectrum:



Quantum algebra:

Claim. If $ba = qab$ then

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k}_q a^k b^{n-k}$$

where

$$(n)_q := 1 + q + \dots + q^{n-1},$$

$$(n)!_q := (1)_q (2)_q \cdots (n)_q,$$

$$\binom{n}{k}_q := \frac{(n)!_q}{(k)!_q (n-k)!_q}.$$

Conjecture: (I. Frenkel, though he may disown this version)

1. Every object in mathematics is the Euler characteristic of a complex.
2. Every operation in mathematics lifts to an operation between complexes.
3. Every identity in mathematics is true up to homotopy at complex-level.



I. Frenkel

The Jones polynomial:

Definition. $\hat{J} : \mathbb{X} \mapsto q(\ -q^2 \curvearrowleft, \quad \hat{J} : \mathbb{X} \mapsto -q^{-2} \curvearrowleft + q^{-1}) \curvearrowright,$

valued in xing-free tangles mod

$$\bigcirc = q + q^{-1}$$

Invariance under R2:

$$\begin{aligned} \hat{J} : \curvearrowleft &\mapsto -q^{-1} \curvearrowleft + \curvearrowright + \bigcirc - q \curvearrowleft \\ &= -q^{-1} \curvearrowleft +) \curvearrowleft + (q + q^{-1}) \curvearrowleft - q \curvearrowleft \\ &=) \curvearrowleft \end{aligned} \quad \square$$

Complexes:

$$\Omega = (\Omega^{-n-} \longrightarrow \Omega^{-n-+1} \longrightarrow \dots \longrightarrow \Omega^{n+})$$

Morphisms:

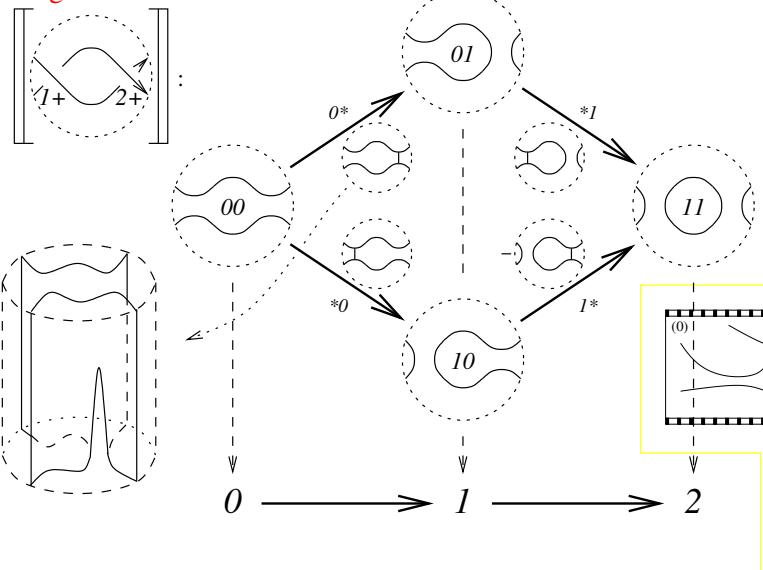
$$\begin{array}{ccccccc} \dots & \longrightarrow & \Omega_0^{r-1} & \xrightarrow{d^{r-1}} & \Omega_0^r & \xrightarrow{d^r} & \Omega_0^{r+1} \longrightarrow \dots \\ & & F^{r-1} \downarrow & & F^r \downarrow & & F^{r+1} \downarrow \\ \dots & \longrightarrow & \Omega_1^{r-1} & \xrightarrow{d^{r-1}} & \Omega_1^r & \xrightarrow{d^r} & \Omega_1^{r+1} \longrightarrow \dots \end{array}$$

Homotopies:

$$\begin{array}{ccccc} \Omega_0^{r-1} & \xrightarrow{d^{r-1}} & \Omega_0^r & \xrightarrow{d^r} & \Omega_0^{r+1} \\ F^{r-1} \downarrow G^{r-1} \quad h^r \swarrow & F^r \downarrow G^r \quad h^{r+1} \swarrow & F^{r+1} \downarrow G^{r+1} \quad \downarrow & & \\ \Omega_1^{r-1} & \xrightarrow{d^{r-1}} & \Omega_1^r & \xrightarrow{d^r} & \Omega_1^{r+1} \end{array}$$

$F^r - G^r = h^{r+1} d^r + d^{r-1} h^r$

Tangles:



All arrows in an arbitrary additive category!

Example:

$$\begin{array}{c} \text{Diagram showing a directed graph of tangles with labels: } \\ \begin{matrix} & q^4(q+q^{-1}) & & q^5(q+q^{-1})^2 & \\ 100 & & & 110 & \\ \downarrow & & & \downarrow & \\ 000 & & 010 & & 101 \\ \downarrow & & \downarrow & & \downarrow \\ 001 & & 011 & & \end{matrix} \\ \text{with edges labeled with polynomials: } \\ \begin{matrix} & q^3(q+q^{-1})^2 & & q^4(q+q^{-1}) & & q^5(q+q^{-1})^2 & & q^6(q+q^{-1})^3 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ q^3(q+q^{-1})^2 & - & 3q^4(q+q^{-1}) & + & 3q^5(q+q^{-1})^2 & - & q^6(q+q^{-1})^3 \end{matrix} \end{array}$$

Mat(C):

$$\begin{array}{ccc} \left(\begin{array}{c} \mathcal{O}'_1 \\ \mathcal{O}''_1 \\ \mathcal{O}'_2 \\ \mathcal{O}''_2 \end{array} \right) & \xrightarrow{\begin{array}{c} G_{11} \\ G_{21} \\ G_{31} \end{array}} & \left(\begin{array}{c} \mathcal{O}'_1 \\ \mathcal{O}'_2 \end{array} \right) \\ & \xrightarrow{\begin{array}{c} F_{21} \\ F_{22} \\ F_{23} \end{array}} & \left(\begin{array}{c} \mathcal{O}_1 \\ \mathcal{O}_2 \end{array} \right) \end{array}$$

Cob:

$$\begin{array}{c} \text{Diagram showing cobordism operations: } \\ \text{Left: } \text{Right: } \\ \text{Diagram showing a cylinder with a handle, followed by a sequence of operations involving handles and strands.} \end{array}$$

Movie moves:

