Pensieve header: The ugly recovery formulas; continues “k=2 Analysis in QU V5.nb” in pensieve://Projects/SL2Portfolio/.

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\HUJI-1912"];
<< KnotTheory`
$k = 2;
```

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

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ToFileName: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}].

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```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[∘]:=
```
𝔼[L_, Q_, P_]$k_ := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
𝔼d_→r_[L_, Q_, P_]$k_ := 𝔼d→r @@ 𝔼[L, Q, P]$k;
```

In[∘]:=
```
𝔼3@𝔼[ω_, L_, Q_, Ps_] := Simplify /@ 𝔼[L, ω^-1 Q, ω^-1 (ω^-4 ϵ)^(-1+Range@Length@Ps).Ps]$k;
𝔼3@𝔼sp___[as___] := 𝔼3@𝔼[as] /. 𝔼 → 𝔼sp;
```

In[∘]:=
```
Clear[QP, ω];
QP[Knot[n_, k_]] := QP[Knot[n, k]] = Module[{fname},
    fname = "../../Projects/SL2Invariant/k=2/Data/" <>
      ToString[n] <> "_" <> ToString[k] <> ".m";
    Collect[𝔼3[Get[fname][[2, 2]]][[3]] // Normal, ϵ, Simplify]
   ];
ω[K_Knot] := ω[K] = Factor[(QP@K /. ϵ → 0)^-1];
ck_,d_[K_Knot] :=
 Factor[SeriesCoefficient[QP[K], {y, 0, 0}, {ϵ, 0, k}, {a, 0, d}] ω[K]^(1+2 k-d)]
```

In[∘]:=
```
p1[K_Knot] := p1[K] = Factor[T (-c1,0[K] + ω[K] T ∂T ω[K]) / (T - 1)^2];
```

In[∘]:=
```
p2[K_Knot] := p2[K] = Expand[-2 c2,0[K] + ω[K] c2,1[K]];
```

In[∘]:= p2[Knot[8, 21]]

Out[∘]=
$$-31226+\frac{3}{T^8}-\frac{28}{T^7}+\frac{49}{T^6}+\frac{352}{T^5}-\frac{2489}{T^4}+\frac{8164}{T^3}-\frac{17530}{T^2}+\frac{27092}{T}+27092\,T-17530\,T^2+8164\,T^3-2489\,T^4+352\,T^5+49\,T^6-28\,T^7+3\,T^8$$

In[∘]:=
```
MyCollect[ℰ_, vs_List] := MyCollect[ℰ, vs, Identity];
MyCollect[ℰ_, vs_List, simp_] :=
  Total[CoefficientRules[ℰ, vs] /. ((ps_ → c_) ⧴ simp[c] Times @@ (vs^ps))];
MyCollect[ℰs_List, vs_List] := MyCollect[#, vs] & /@ ℰs;
MyCollect[ℰs_List, vs_List, simp_] := MyCollect[#, vs, simp] & /@ ℰs;
MyCollect[sd_SeriesData, vs_List] := MapAt[MyCollect[#, vs] &, sd, 3];
MyCollect[sd_SeriesData, vs_List, simp_] := MapAt[MyCollect[#, vs, simp] &, sd, 3];
```

In[∘]:=
$$\text{RecoveryFormula} = \omega^{-1} + \left(\frac{-2\,T\,\omega\,d\omega}{(T-1)}\,x\,y + 2\,T\,\omega\,d\omega\,a + \left(\omega\,T\,d\omega - \frac{(T-1)^2}{T}\,p1\right)\right)\frac{\epsilon}{\omega^3} +$$
$$\Bigg(2\,T\,\omega^2\left(2\,d\omega^2\,T - d\omega\,\omega - dd\omega\,T\,\omega\right)a^2 + \Bigg(\frac{2\,(-1+T)\,p1\,\left((1+T)\,\omega - 3\,(-1+T)\,T\,d\omega\right)}{T} +$$
$$2\,\omega\left((-1+T)^2\,dp1 + 2\,T^2\,(d\omega)^2 - T\,\omega\,(d\omega + T\,dd\omega)\right)\Bigg)\,\omega\,a +$$
$$\frac{T\,\omega^2\left(4\,d\omega^2\,(-1+T)\,T - d\omega\,(-3+T)\,\omega - 2\,dd\omega\,(-1+T)\,T\,\omega\right)}{(-1+T)^3}\,x^2\,y^2 -$$
$$\frac{2\,\omega\left(-3\,d\omega\,p1\,(-1+T)\,T + dp1\,(-1+T)\,T\,\omega + p1\,(1+T)\,\omega\right)}{T}\,x\,y +$$
$$\frac{4\,T\,\omega^2\left(2\,d\omega^2\,(1-T)\,T - d\omega\,\omega - dd\omega\,(1-T)\,T\,\omega\right)}{(-1+T)^2}\,a\,x\,y - \Bigg(\frac{p2}{2} + \omega\Bigg(3\,d\omega\,p1\,(-1+T)^2 -$$
$$\frac{\left(p1\,(-1+T^2) + T\left(dp1\,(-1+T)^2 + 2\,d\omega^2\,T^2\right)\right)\omega}{T} + T\,(d\omega + dd\omega\,T)\,\omega^2\Bigg)\Bigg)\Bigg)\frac{\epsilon^2}{\omega^5};$$

In[∘]:=
```
Monitor[Union@Table[
    Simplify[r = -QP[K] + RecoveryFormula /.
        {ω → ω[K], dω → ∂_T ω[K], ddω → ∂_{T,T} ω[K], p1 → p1[K], dp1 → ∂_T p1[K], p2 → p2[K]}],
    {K, AllKnots[{3, 10}]}],
  K]
```

Out[∘]= {0}

In[∘]:=
```
MyCollect[Log[(ω RecoveryFormula /. ϵ → ω^2 ϵ) + O[ϵ]^3], {a, x, y}, FullSimplify]
```

Out[∘]=

$$\left(-\frac{p1\,(-1+T)^2}{T}+d\omega\,T\,\omega+2\,a\,d\omega\,T\,\omega-\frac{2\,d\omega\,T\,x\,y\,\omega}{-1+T}\right)\epsilon+\Bigg(-2\,a^2\,T\,\omega^2\,\left(-d\omega^2\,T+d\omega\,\omega+dd\omega\,T\,\omega\right)+$$
$$\frac{T\,x^2\,y^2\,\omega^2\,\left(2\,d\omega^2\,(-1+T)\,T-d\omega\,(-3+T)\,\omega-2\,dd\omega\,(-1+T)\,T\,\omega\right)}{(-1+T)^3}-$$
$$\frac{4\,a\,T\,x\,y\,\omega^2\,\left(d\omega^2\,(-1+T)\,T+d\omega\,\omega-dd\omega\,(-1+T)\,T\,\omega\right)}{(-1+T)^2}+$$
$$2\,x\,y\,\omega\left(2\,d\omega\,p1\,(-1+T)+\frac{d\omega^2\,T^2\,\omega}{-1+T}-\frac{\left(dp1\,(-1+T)\,T+p1\,(1+T)\right)\omega}{T}\right)+$$
$$\frac{2\,a\,\omega\left(-2\,d\omega\,p1\,(-1+T)^2\,T+p1\,(-1+T^2)\,\omega+T\left(dp1\,(-1+T)^2+d\omega^2\,T^2\right)\omega-T^2\,(d\omega+dd\omega\,T)\,\omega^2\right)}{T}+$$
$$\frac{1}{2\,T^2}\Big(-p1^2\,(-1+T)^4-2\,p1\,(-1+T)\,T\,\omega\,\left(2\,d\omega\,(-1+T)\,T-(1+T)\,\omega\right)+$$
$$T^2\left(-p2+\omega^2\left(2\,dp1\,(-1+T)^2+T\left(3\,d\omega^2\,T-2\,(d\omega+dd\omega\,T)\,\omega\right)\right)\right)\Big)\Bigg)\epsilon^2+O[\epsilon]^3$$

In[∘]:=
```
mencode = {dω → A, ddω → B, p1 → p₁, dp1 → D, p2 → p₂};
tencode = {"A" → "\\dot{\\omega}", "B" → "\\ddot{\\omega}", "D" → "\\dot{p}_1"};
```

C_{kij} is the coefficient of $\epsilon^k a^i (xy)^j$ in Log[(ω RecoveryFormula /. $\epsilon \to \omega^2 \epsilon$) + $O[\epsilon]^3$]; T_{kij} is its TeXForm:

In[∘]:=
```
T100 = ToString@TeXForm[(C100 = HoldForm[T ω dω - p1 (T - 1)^2 / T]) /. mencode];
T110 = ToString@TeXForm[(C110 = HoldForm[2 T ω dω]) /. mencode];
T101 = ToString@TeXForm[(C101 = HoldForm[2 T ω dω / (1 - T)]) /. mencode];
If[
 Simplify[SeriesCoefficient[Log[(ω RecoveryFormula /. ϵ → ω^2 ϵ) + O[ϵ]^3], 1] ==
   ReleaseHold[C100] + ReleaseHold[C110] a + ReleaseHold[C101] x y],
 tex = ToString@StringReplace["\\[
  P^{(1)} =
    \\left(T100 \\right)
    + T110 a
    + T101 xy,
\\]
",
    {"T100" → T100, "T110" → T110, "T101" → T101}];
 tex = StringReplace[tex, tencode];
 DeleteFile["P1.tex"];
 WriteString["P1.tex", tex];
 Close["P1.tex"];
 tex
]
```

Out[∘]=
```
\[
  P^{(1)} =
    \left(T \omega  \dot{\omega}-\frac{p_1 (T-1)^2}{T} \right)
    + 2 T \omega  \dot{\omega} a
    + \frac{2 T \omega  \dot{\omega}}{1-T} xy,
\]
```

In[∘]:=
```
T200 = ToString@
   TeXForm[(C200 = HoldForm[1/(2 T^2) (T^2 (ω^2 (2 dp1 (T - 1)^2 + T (3 dω^2 T - 2 (dω + ddω T) ω)) - p2) -
         p1^2 (T - 1)^4 - 2 p1 (T - 1) T ω (2 dω (T - 1) T - (1 + T) ω))]) /. mencode];

T210 = ToString@TeXForm[(C210 = HoldForm[
        (2 ω (p1 (T^2 - 1) ω - 2 dω p1 (T - 1)^2 T + T (dp1 (T - 1)^2 + dω^2 T^2) ω - T^2 (dω + ddω T) ω^2))/T]) /.
     mencode];
T220 = ToString@TeXForm[(C220 = HoldForm[2 T ω^2 (dω^2 T - dω ω - ddω T ω)]) /. mencode];
T201 = ToString@
   TeXForm[(C201 = HoldForm[2 ω (2 dω p1 (T - 1) + (dω^2 T^2 ω)/(T - 1) - ((dp1 (T - 1) T + p1 (1 + T)) ω)/T)]) /.
     mencode];
T211 = ToString@TeXForm[(C211 = HoldForm[(4 T ω^2 (ddω (T - 1) T ω - dω^2 (T - 1) T - dω ω))/(T - 1)^2]) /.
     mencode];
T202 = ToString@TeXForm[(C202 = HoldForm[
        (T ω^2 (2 dω^2 (T - 1) T - dω (T - 3) ω - 2 ddω (T - 1) T ω))/(T - 1)^3]) /. mencode];
If[
 Simplify[SeriesCoefficient[Log[(ω RecoveryFormula /. ϵ → ω^2 ϵ) + O[ϵ]^3], 2] ==
   ReleaseHold[C200] + ReleaseHold[C210] a + ReleaseHold[C220] a^2 +
    ReleaseHold[C201] x y + ReleaseHold[C211] a x y + ReleaseHold[C202] x^2 y^2],
 tex = ToString@StringReplace["\\begin{multline*} \\scriptstyle
  P^{(2)} =
    T200 \\\\ \\scriptstyle
    + T210 a \\\\ \\scriptstyle
    + T220 a^2
    + T201 x y \\\\ \\scriptstyle
    + T211 a x y
    + T202 x^2y^2.
\\end{multline*}",
    {"T200" → T200, "T210" → T210, "T220" → T220, "T201" → T201, "T211" → T211, "T202" → T202}];
 tex = StringReplace[tex, tencode];
 DeleteFile["P2.tex"];
 WriteString["P2.tex", tex];
 Close["P2.tex"];
 tex
]
```

Out[ ]=
```
\begin{multline*} \scriptstyle
  P^{(2)} =
    \frac{T^2 \left(\omega ^2 \left(2 \dot{p}_1 (T-1)^2+T \left(3 \dot{\omega}^2 T-2
  (\dot{\omega}+\ddot{\omega} T) \omega \right)\right)-p_2\right)-p_1^2 (T-1)^4-2
  p_1 (T-1) T \omega  (2 \dot{\omega} (T-1) T-(1+T) \omega )}{2 T^2} \\ \scriptstyle
    + \frac{2 \omega  \left(p_1 \left(T^2-1\right) \omega -2 \dot{\omega}
  p_1 (T-1)^2 T+T \left(\dot{p}_1 (T-1)^2+\dot{\omega}^2 T^2\right) \omega
  -T^2 (\dot{\omega}+\ddot{\omega} T) \omega ^2\right)}{T} a \\ \scriptstyle
    + 2 T \omega ^2 \left(\dot{\omega}^2 T-\dot{\omega}
  \omega -\ddot{\omega} T \omega \right) a^2
    + 2 \omega  \left(2 \dot{\omega} p_1 (T-1)+\frac{\dot{\omega}^2
  T^2 \omega }{T-1}-\frac{\left(\dot{p}_1 (T-1) T+p_1
  (1+T)\right) \omega }{T}\right) x y \\ \scriptstyle
    + \frac{4 T \omega ^2 \left(\ddot{\omega} (T-1) T \omega
  -\dot{\omega}^2 (T-1) T-\dot{\omega} \omega \right)}{(T-1)^2} a x y
    + \frac{T \omega ^2 \left(2 \dot{\omega}^2 (T-1) T-\dot{\omega}
  (T-3) \omega -2 \ddot{\omega} (T-1) T \omega \right)}{(T-1)^3} x^2y^2.
\end{multline*}
```