The Big Bang

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0. Introduction

What is the nature of the universe? Don't feel bad, I don't know either and I don't think anyone will ever know. However, if we ask ourselves the less ambitious question: "What is the structure of the universe on a cosmological scale?" then Einstein's theory of General Relativity (GR) can be used to explore its answer. Can it be used to answer it definitely? No. At present, our inability to accurately measure certain cosmological parameters prevents us from knowing exactly what type of universe we live in. Also, GR predicts that there are so-called *dark matter* regions of the universe. Dark matter is simply matter that emits almost nothing, like a black hole, so it is difficult to directly observe such regions. It is still possible to detect some types of dark matter, for example by observing the bending of light rays or Hawking radiation, but we cannot detect all of it. It will turn out that our theory is sensitive to the amount of matter in the present universe, so the problem of undetectable matter again prevents us from knowing for sure what possible universe we live in. However, there is one thing our theory predicts almost certainly occurred: an initial singularity, the *Big Bang*.

Despite these difficulties, with a single philosophically motivated and observationally verified assumption, GR can be used to set up possible models of the universe that agree closely with the currently observed universe. These models explain two observed mysteries: the 3K cosmic microwave background and apparent over abundance of ^{2}H. The explanation of these phenomena are two of the great successes of the theory. In return, observation of these two phenomena indicate that a big bang must have occurred, even if our model is not quite correct. We will now make it our task to come up with a model of the universe, i.e. a solution to the field equations, and explore its properties.

1. The Copernican Principle

In the earliest days man placed himself at the centre of the universe. Copernicus changed this view. Since then we have come to believe that Earth is just a regular planet orbiting an average start in an average galaxy that is part of an average cluster, etc. This view is known as the *Copernican principle*, and shall be our only assumption. This leads to the ideas of *spatial homogeneity* and *isotropy*. Spatial homogeneity means that all points in the universe are equivalent, i.e. there is no preferred location in the universe. Isotropy means that at any point the universe looks the same in all directions, i.e. there is no preferred direction. These two ideas can be made mathematically rigorous. Obviously a space-time is isotropic if and only if it is spherically symmetric about every point. The idea of spatial homogeneity is somewhat more technical (we follow the conventions of Hawking and Ellis). At first we might think a space-time is spatially homogeneous if and only if we can transform any point into any other, i.e. if and only if there is a group of isometries such that for any two points we can find an isometry that maps the first point into the second (this is equivalent to what Carroll calls *maximally symmetric spaces*). This is not quite right. Spatial homogeneity refers only to the spatial part of space-time. Thus we say a space-time is spatially homogeneous if and only if it admits a group of isometries whose surfaces of transitivity are space-like three-surfaces. This means that for any two points on the same space-like three-surface we can always find an isometry that maps the first point into the second.

It is hard to experimentally verify the extent to which the universe is homogeneous. This is due to the difficulty of measuring the separation between distant objects and ourselves. We can, however, easily measure the isotropy about us. We simply make extragalactic observations in many directions and see if they are all the same. We can also measure the isotropy of the cosmic microwave background. Both of these observations lead to the conclusion that the universe is highly isotropic about us (according to Carroll the cosmic microwave background deviates from regularity on the order of 10^{-5} or less).

We will accept the Copernican principle, i.e. we assume the universe is homogeneous. Of course the universe is not exactly homogeneous, the universe inside a black hole presumably does not resemble the universe we observe here on Earth in any way. We only assume it holds on the very largest scales. Combining this with the fact that the universe seems to be isotropic about us, we conclude that the universe is isotropic about every point.

2. Robertson-Walker Spaces

As you might expect, it can be shown that isotropy about every point implies homogeneity (the converse however, is not true, for example a homogeneous vacuum with a constant electric field). Thus it is enough to assume isotropy at each point. When we look at distant galaxies we see that they are moving away from us, thus we only conclude that the universe is isotropic (and hence homogeneous) in space but not in time. It can be shown (first by Walker in 1944) that such a space-time admits a six-parameter group of isometries whose surfaces of transitivity are space-like three-surfaces of constant curvature. Spaces with this property are called *Robertson-Walker spaces*. (Minkowski, de Sitter and anti-de Sitter space are all examples of the more generic R-W space, and R-W space is an example of the more generic spatially homogeneous space.) All of this is quite technical. We will now make these abstract notions concrete by introducing the space-time manifold and the metric (following section 8 of Carroll's notes).

What we have discussed above implies that space-time has the structure of $R \times \Sigma$ where R (the real line) represents the time direction and Σ is a homogeneous and isotropic three-dimensional manifold. The metric can be written

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}(u)du^i du^j \qquad (2.1)$$

where t is the time-like coordinate, (u^1, u^2, u^3) are coordinates on Σ, and γ_{ij} is the homogeneous and isotropic metric on Σ. The function $a(t)$ is called the scale factor. Physically it tells us how big the space-like slice Σ is at the time t. These coordinates are an example of a type of coordinate system known as *comoving coordinates*. A comoving coordinate system is one where the metric has no cross terms between the time-like and space-like coordinates and the space-like components are all proportional to a single function of t. An observer with constant u^i is called a “comoving observer”. Only comoving observers think the universe is isotropic. Physically we think of galaxies as comoving. Earth is not quite comoving due to its orbit around our galaxy. We can, in fact, detect a small dipole anisotropy in the cosmic microwave background due to the conventional Doppler effect. As was mentioned above, it is observed that galaxies are moving away from our own. It is also observed that galaxies are moving away from each other. In our model this corresponds to a “lengthening of the metric” due to the scale factor. This is why we say the universe is “expanding”. Galaxies are not “flying” away from each other, the space between them is expanding due to the scale factor. It is this lengthening of the metric that is also responsible for the *redshift*, i.e. the drop in frequency of a photon between emission and absorption, not the Doppler effect.

We now try to find an explicit formula for the metric. For that we will need to choose a specific coordinate system. Before we do that we can write down some properties that any maximally symmetric metric has. Let $\overline{R}_{ijjkl}$ be the curvature tensor associated with the three-metric γ_{ij}. Then it can be shown that the maximal symmetry of γ_{ij} implies

$$\overline{R}_{ijkl} = k(\gamma_{ij}\gamma_{jl} - \gamma_{il}\gamma_{jk}), \quad (2.2)$$

where k is some real constant (see Wald p.93-94 for the sickening details). After a simple calculation the associated Ricci tensor is then found to be

$$\overline{R}_{jl} = 2k\gamma_{jl}. \quad (2.3)$$

Since Σ is isotropic it is spherically symmetric about every point. Thus we can choose any point and set up spherical polar coordinates around it and use a metric that is spherically symmetric about it, i.e. we can use the Schwarzchild metric centred at the chosen point. Following calculations similar to the ones we did in class we can put the metric into the following form:

$$\gamma_{ij}du^i du^j = e^{2\beta(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2.4)$$

Setting the coefficients of the Ricci tensor obtained from (2.4) equal to those given by (2.3) allows us to solve for $\beta(r)$. We get

$$\beta(r) = -\frac{1}{2}\ln(1 - kr^2). \quad (2.5)$$

This gives us the following metric on the total space-time

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \qquad (2.6)$$

This is the famous *Robertson-Walker metric*. It is the metric associated with R-W spaces. Notice that the metric is invariant under the simultaneous substitution

$$\begin{aligned} k &\mapsto \frac{k}{|k|} \\ r &\mapsto \sqrt{|k|}r \qquad (2.7) \\ a &\mapsto \frac{a}{\sqrt{|k|}}. \end{aligned}$$

Thus the only important parameter is $k/|k|$, so we may assume $k=1$, $k=0$, or $k=-1$.

When $k=0$ Σ is flat. The metric on Σ can then be written as

$$dr^2 + r^2 d\Omega^2 = dx^2 + dy^2 + dz^2 \qquad (2.8)$$

which is just the metric on Euclidean three-space. When $k=1$ Σ is a surface of constant positive curvature. It looks as though the metric is singular at $r=\pm 1$. This is obviously due to our choice of coordinates. If we let $r=\sin\chi$ then the metric on Σ can be written as

$$d\chi^2 + \sin^2\chi d\Omega^2 \qquad (2.9)$$

which is just the metric of a three-sphere. When $k=-1$ Σ is a surface of constant negative curvature. If we let $r=\sinh\psi$ then the metric on Σ can be written as

$$d\psi^2 + \sinh^2\psi d\Omega^2 \qquad (2.10)$$

which is the metric of a three-hyperboloid.

We see that topologically there are two cases. The case of a three-sphere is the most interesting. It is compact and hence represents a universe that is finite but has no boundary. For this reason we say it is closed. The other two cases are called open since there are infinite. Actually, by making certain topological identifications the two open cases can be made closed but the identifications seem to be unnatural. So an interesting question is whether or not our universe is open or closed.

Since we have the metric (2.6) we can easily compute the Christoffel symbols, the curvature and Ricci tensors, the Ricci scalar, as well as the Einstein tensor. The results are summarized on p.220 of Carroll's notes, equations (8.12)-(8.14) and p.97 of Wald, equations (5.2.3)-(5.2.12).

3. Cosmic Dust

In order to explore the dynamical behaviour of the universe we will need its stress-energy-momentum tensor $T_{\mu\nu}$. This is obviously an extremely complicated object. It depends on space and time and takes into account every movement of every particle and the change in every field in the universe; it is even affected by your movements! There is no way we could ever write down such an entity. It is time to make some more approximations. Contrary to what some people believe, on the cosmological scale nothing we do affects the universe. In fact on such a large scale our planet, solar system, and galaxy play no important role in the dynamics of the universe. This is due the vastness of our universe. Ignoring our galaxy would be like ignoring an atom that is part of the ocean. One may then think that nothing affects the dynamics of the universe and hence we may assume the universe is empty. This leads to the vacuum equations, which would give a static universe. As was mentioned earlier, it is observed that the universe is in fact not static, it appears to be expanding. It is believed that most of the mass and energy of the universe is concentrated in ordinary matter in galaxies, but this is not certain. Thus we make the following first approximation. We will the model the contents of the universe as a *perfect fluid*. A perfect fluid is defined as one that is isotropic in its rest frame. In the present epoch of the universe we could even model the contents of the universe as *dust* (a perfect fluid with no pressure) where the particles of dust are galaxies. Other forms of energy, such as the cosmic microwave background, seem to be negligible. However, we will find that these other forms of energy will make a significant contribution in earlier eras so we shall not make this further assumption.

The stress-energy-momentum tensor for a perfect fluid is

$$T_{\mu\nu} = (p+\rho)U_\mu U_\nu + g_{\mu\nu}\,, \qquad (3.1)$$

where ρ is the energy (mass) density and p is the pressure, both measured in the fluid's rest frame, and U^μ is the fluid's four-velocity. You may wonder why this is the correct tensor, as I did. Chapter 4 of Schutz gives a clear and thorough discussion; I will only summarize its properties and hopefully this convinces you that it really is the correct tensor. It can be shown that the time-time component gives the energy density (i.e. ρ), the space-time components give the energy fluxes, and the space-space components give the momentum fluxes. It can also be shown that the conservation law, i.e. the vanishing of the divergence, agrees with the conventional mass-energy conservation laws as well as with the laws of thermodynamics. In fact, it can be shown that this is the most general rank-2 tensor that is homogeneous and istropic that only depends on the energy and pressure, so no harm is done by this assumption. Since the fluid is isotropic in its rest frame this must coincide with the comoving frame, so the fluid is at rest in comoving coordinates. Its four-velocity in this frame is then

$$U^\mu = (1,0,0,0)\,. \qquad (3.2)$$

Raising one index on $T_{\mu\nu}$ gives

$$T^\mu{}_\nu = diag(-\rho, p, p, p)\,, \qquad (3.3)$$

with trace

$$T = T^\mu{}_\mu = -\rho + 3p\,. \qquad (3.4)$$

Now that we have a complete model of the universe we can investigate its dynamics, as governed by Einstein's theory of general relativity. First we will look at the first component of the energy conservation law. After some calculations we get:

$$0 = \nabla_\mu T^\mu{}_0 = -\partial_0\rho - 3\frac{\dot{a}}{a}(\rho + p)\,. \qquad (3.5)$$

(The a and $\dot{a}$ are introduced through the Christoffel symbols.) To go further we need to choose a relationship between ρ and p. For our discussion we need two. If $p = 0$ then the fluid is called dust. As mentioned, the dust is ordinary matter, concentrated in galaxies. We call a universe whose energy is mostly due to dust *matter-dominated*. In this case we can integrate (3.5) and obtain

$$\rho \propto a^{-3}. \qquad (3.6)$$

This has a simple physical interpretation. As the universe expands (or contracts) the number of dust particles, i.e. the amount of energy-mass, is conserved, hence the energy density decreases (or increases). If $p = \rho/3$ then the fluid is called *radiation*. It may describe actual EM radiation or relativistic particles that behave as radiation. We say a universe in which most of the energy is in this form is *radiation-dominated*. This relation comes from matching the expression for $T^{\mu\nu}$ in terms of the field strength $F^{\mu\nu}$ of the radiation with that given by (3.1) (see p.224-225 of Carroll's notes). In this case, integrating (3.5) gives

$$\rho \propto a^{-4}. \qquad (3.7)$$

Again we have the same simple physical interpretation except this time the energy density falls off faster. It turns out this additional loss is due to the energy lost by photons when they redshift. It is believed that currently the universe is matter-dominated, with $\rho_{mat} / \rho_{rad} \approx 10^6$, but this is uncertain due to the unknown amount of dark matter in the universe. In the past when the universe was much smaller it would have been radiation-dominated.

4. Friedmann-Robertson-Walker Cosmology

So far we have not made use of Einstein's equations. These will tell us the behaviour of the scale factor $a(t)$, i.e. they control the dynamics of the universe. It is more convenient for us to write the field equations in the following equivalent form

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right). \qquad (4.1)$$

We use this form since the trace T is much simpler than R. After a messy calculation we find that for the R-W metric (2.6)

$$R_{00} = -3\frac{\ddot{a}}{a}. \quad (4.2)$$

Equating this with the left side of (4.1) we find

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \quad (4.3)$$

By similar (even messier) means we find the space-space equations all give the following result (this is due to isotropy):

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G(\rho - p). \quad (4.4)$$

Since $R_{\mu\nu}$ and $T_{\mu\nu}$ are diagonal the time-space equations tell us $0 = 0$, hopefully something we already knew. Combining (4.3) and (4.4) allows us to eliminate the second derivative and obtain the following result:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \quad (4.5)$$

Together (4.3) and (4.5) are known as the *Friedmann equations*. Any R-W space whose metric obeys the Friedmann equations is known as a Friedmann-Robertson-Walker universe. (Friedmann was the first to investigate the dust filled closed universe in 1922.)

We will now define some cosmological parameters to help us interpret physically what the Friedmann equations imply. The *Hubble parameter* is defined as

$$H = \frac{\dot{a}}{a}. \quad (4.6)$$

Physically it tells us the rate of the expansion of the universe (we need to divide by a to get a physically measurable quantity). The current value of H, denoted H_0, is known as the Hubble constant (after Hubble who, along with Slipher, were the first to observe that galaxies are moving away from each other). The *deceleration parameter* is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (4.7)$$

It measures the rate of change of the rate of expansion. The *density parameter* is defined as

$$\Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_{crit}} \qquad (4.8)$$

where

$$\rho_{crit} = \frac{3H^2}{8\pi G} \qquad (4.9)$$

is the *critical density*. Note that in general all these quantities change in time. With these definitions the Friedmann equation (4.5) can be written

$$\Omega - 1 = \frac{k}{H^2 a^2}. \qquad (4.10)$$

Thus we can determine the sign of k be measuring Ω:

$$\rho < \rho_{crit} \leftrightarrow \Omega < 1 \leftrightarrow k = -1 \leftrightarrow open$$
$$\rho = \rho_{crit} \leftrightarrow \Omega = 1 \leftrightarrow k = 0 \leftrightarrow flat$$
$$\rho > \rho_{crit} \leftrightarrow \Omega > 1 \leftrightarrow k = 1 \leftrightarrow closed.$$

So the density parameter tells us which of the three universes we live in. Measuring it experimentally is currently a major area of research.

5. The Big Bang

Rather than solve the Friedmann equations exactly we will investigate their general qualitative behaviours. We will assume the universe is filled with a perfect fluid with positive energy ($\rho > 0$) and nonnegative pressure ($p \geq 0$). By (4.3) we have $\ddot{a} < 0$. Since we know the universe is expanding this means the expansion is "decelerating". Intuitively, it is the gravitational attraction of the matter in the universe that is slowing this expansion. Since under this assumption the universe has always been decelerating, it must have been expanding even faster in the past. If we go back far enough we always reach an initial singularity ($a = 0$). Note that if $\ddot{a} = 0$ then $a(t)$ is a straight line and H_0^{-1} would be the age of the universe. Since the universe has been decelerating its actual age is less than this.

The singularity $a = 0$ is the Big Bang. It can be shown that all world-lines intersect at this point and the energy density becomes infinite. It is much worse than this, space-time itself is singular at this point, hence no known physical laws could possibly hold at this time. It does not represent the explosion of matter into an empty universe; it is the creation of the universe from a singular point. Because the density and curvature were so large near the big bang it is not expected that classical general relativity gives an accurate description of this era. The effects of quantum mechanics and perhaps some other unknown laws are expected to make significant contributions at this time. No current physical theory can reliably describe this era and a large amount of current research is dedicated to finding quantum theories of gravity that will hopefully help us understand it. It is safe to say that currently we know almost nothing about it.

One may think that the fact that the universe is not perfectly homogeneous and isotropic would allow us to avoid this singularity. This is not the case. We could first drop isotropy and only require homogeneity. It can be shown (see Hawking and Ellis §5.4) that spatially homogeneous cosmologies with $\rho > 0$ and $p \geq 0$ necessarily have singularities. By thinking of the Big Bang as the time reverse of a gravitational collapse, the singularity theorems of Hawking and Ellis can be used to show that our universe almost certainly had singularities in its past, although the nature of these singularities cannot be explained by these theorems. So, although we know almost nothing about the Big Bang we can say theoretically it is almost certain that this is how the universe began.

Experimentally what can we say? The observed redshift of distant galaxies is strong evidence that the universe is currently expanding. There is also the mysterious cosmic microwave background and over abundance of ^2H. It seems the only explanation of both of these is that at some point in the past the universe was very hot and dense. Thus we can conclude that even if there was no Big Bang the universe did contract to a small size in its past, whether or not quantum effects were able to prevent the initial singularity is still an open question.

5. Life after a Big Bang

Using the Friedmann equation (4.5) we can find the behavior of $a(t)$ for the different cases $k=+1,0,-1$. If $k=-1,0$ then we have

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 + |k|. \qquad (5.1)$$

We are assuming $\rho > 0$ so the right hand side is strictly positive. Hence $\dot{a}$ is never zero and so must be always positive or always negative. Since we know that it is currently positive it must have always been and continue to be so. We see that open and flat universes expand forever; they are temporally as well as spatially open. However, if we assume a negative or zero energy then it is possible to have open universes that are not temporally open. Recall that in a matter-dominated universe the quantity ρa^3 is constant. In general we have (using energy conservation (3.5)):

$$\frac{d}{dt}(\rho a^3) = a^3\left(\dot{\rho} + 3\rho\frac{\dot{a}}{a}\right) = -3pa^2\dot{a}. \qquad (5.2)$$

The right hand side is zero or negative (as long as $p \geq 0$). This implies that ρa^2 goes to zero. Looking back at (5.1) we see that this means

$$\dot{a}^2 \to |k|. \qquad (5.3)$$

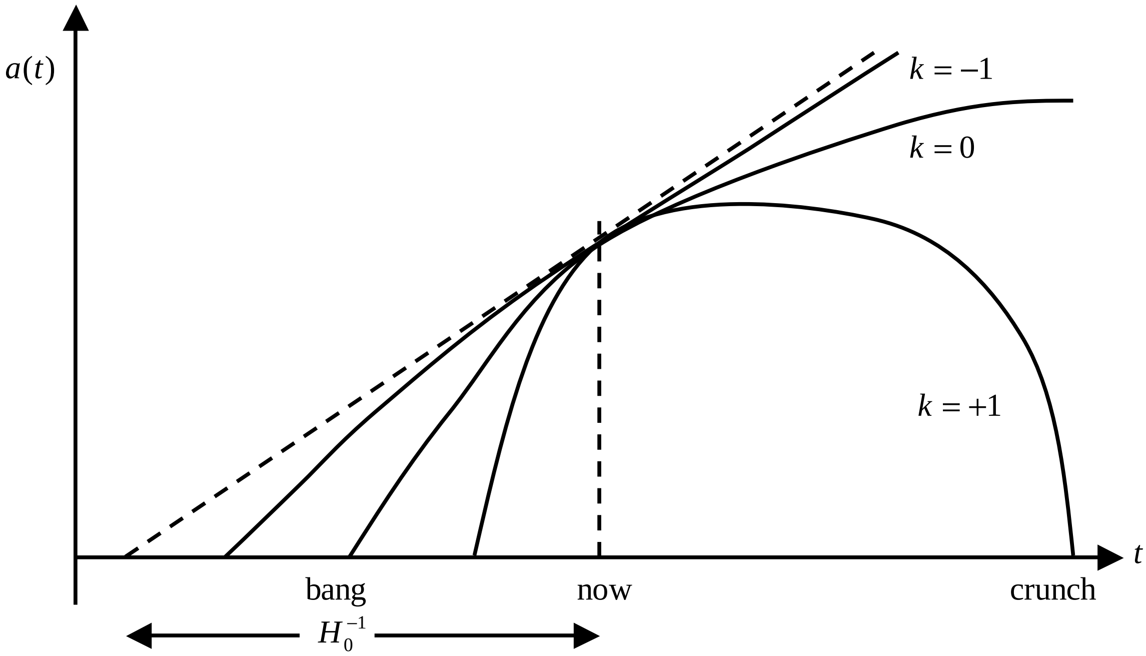
Thus, if $k=-1$ then the rate of expansion approaches the limiting value 1. If $k=0$ then the universe continues to expand but at a slower and slower rate.

For $k=+1$ we have

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - 1. \qquad (5.4)$$

If we still had $a \to \infty$ then $\rho a^2 \to 0$ would imply the right hand side of (5.4) is negative which cannot happen. Thus in this case the universe reaches a maximum size $a_{\max}$ and then starts decreasing. Since $\ddot{a} < 0$ we must have $a = 0$ again for some time in our future, this is known as the *Big Crunch*. Thus spatially closed universes are also closed in time (assuming a positive energy and nonnegative pressure).

Our results can be summarized in the following picture:



The exact solutions for matter filled and radiation filled universes are discussed on p.226-227 of Carroll's notes and summarized in table 5.1, p.98, of Wald.

7. The Cosmological Constant

As we just saw all of our models of the universe are dynamic. Before the observed expansion of the universe in 1929 it was believed that the universe was static. Einstein subsequently proposed a modification to the field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad (7.1)$$

where Λ is a new fundamental constant of nature, the cosmological constant. It can be shown that the new left hand side is in fact the most general two index, symmetric, divergence free tensor that can be constructed solely from the metric and its first two derivatives, so it is natural to add it and try to experimentally determine its value. The new Friedman equations admit a static solution in which ρ, p, and Λ are each positive (the original one did only if $p < 0$ which is not the case for most of the known matter in the universe), it is known as the *Einstein static solution*. Although this solution is static, it is unstable. All three parameters must be fine tuned, and any deviation from these sends the solution away from its static state. After the expansion was observed it was thought the cosmological constant was no longer needed and was for the most part set to zero. Einstein then stated that its creation was his greatest regret.

Recently, the cosmological constant has made a comeback, this time in the form of the vacuum energy. However, this interpretation, originating in particle theory, is itself puzzling. The constant is expected to be much larger than current experimentally determined upper bound. The new equations are equivalent to the original ones if we set

$$T_{\mu\nu}^{(vac)} = -\frac{\Lambda}{8\pi G} g_{\mu\nu} \qquad (7.2)$$

to be the energy of the vacuum. This has the form of a perfect fluid with $\rho = -p$. Looking back at the energy conservation equitation (3.5) we see this implies the vacuum energy density is independent of a, as should be expected. Since the energy density of matter and radiation decreases as the universe expands this vacuum energy will tend to dominate as the universe expands, we say a universe in such a state is *vacuum-dominated*. Since the matter and radiation terms dominate when the universe is dense, the vacuum energy has little effect on the Big Bang and cannot be used to try and avoid it.

Carroll's article *The Cosmological Constant* contains much information on this topic. For a concise description of cosmology with a cosmological constant see p.139 of Hawking and Ellis. It turns out that vacuum only universes have only two exact solutions, the famous de Sitter and anti-de Sitter space for $\Lambda > 0$ and $\Lambda < 0$ respectively.

8. Conclusions

We found that the universe almost certainly began with a big bang. This led us to more interesting questions. Which universe do we live in, i.e. will our universe expand forever or collapse in the big crunch scenario? How old is our universe? A lot of research is going into answering these questions and has resulted in a lot of controversy. Estimates of the Hubble constant are between 50 and 100 km/s/Mpc where 1 Mpc = 1 megaparsec $\approx 3\times 10^{19}$ km. Estimates are obtained by measuring the redshift of distant light sources and it is hard to accurately measure such large distances. That is why there is such a large error in the estimate. It is believed that most of the energy in the universe is concentrated in ordinary matter in galaxies. By estimating the mass density of the universe an estimate of 0.04 for Ω is obtained. Thus it appears that our universe is open. However, recently it has been suggested that the amount of dark matter in the universe could be much higher than originally thought, this would bring Ω closer to 1. An interesting question is why we seem to be so close to the critical value. If we assume that the universe was matter-dominated for most of its existence and $k = 0$ we find that

$$t_{crit} = \frac{2}{3H} \approx 13 \text{ billion years.} \qquad (8.1)$$

If the universe is closed then its age will be smaller than this and if it is open it will be larger. We can also estimate the age of the universe from the age of the oldest known objects, globular star clusters. Estimates by this method put the age of the universe between 10 and 20 billion years. The closeness of this to the critical age is more evidence that the universe really did start with a big bang, however the uncertainty does not allow us to conclude whether or not the universe is open or closed.

Einstein's theory has predicted a very interesting universe; far different from the conventional one we were thought to live in less than a hundred years ago. By trying to answer one question it has led to many interesting unanswered questions. Perhaps its most important feature is its ability to explain two observed phenomena that apparently no other theory of space and time significantly different than Einstein's can account for, the cosmic background radiation and the over abundance of ^{2}H, relics of the Big Bang.

9. References

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