# MAT 137Y: Calculus with proofs
# Test 4 - Part B
# Comments and common errors

## Q1

- The behaviour of powers of $x$ is different as $x \to \infty$ and as $x \to 0^+$:

$$\lim_{x\to\infty} \frac{x + 2x^2}{3x + 5x^5} = \frac{2}{5}, \qquad \text{but} \qquad \lim_{x\to 0^+} \frac{x + 2x^2}{3x + 5x^5} = \frac{1}{3}$$

  In other words, in the sum $x^a + x^{a+b}$ ...

  - ... the "dominant" term is $x^{a+b}$ as $x \to \infty$
  - ... the "dominant" term is $x^a$ as $x \to 0^+$

- It is not enough to prove the integral is convergent for some values of $a$ and $b$.

  You also need to prove that it isn't convergent for the rest of the values.

- When using comparison tests to compare two improper integrals, you do not always get an "if and only if".

  If you use LCT (and the limit exists and is not 0), the first integral is convergent if and only if the second integral is convergent.

  However, if you use

  - BCT, or
  - LCT with limit 0 or $\infty$

  you only get one implication.

- If you split an improper integral as a sum of two improper integrals, you need to verify that they converge separately.

## Q2

- This proof is extremely similar to the one in Video 11.5. You only need to modify a few things. You won't get any credit for copying down the proof of Video 11.5. You only get credit for understanding what needs to be modified and *writing it correctly*. This means you could write a proof that is "partially correct" and still get 0 points.

- If you write something like...

  "*The sequence is divergent to* $\infty$ *so*

$$\forall M \in \mathbb{R},\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N}, \quad n \geq n_0 \implies a_n \geq M$$

  "*Take* $A = \min\{a_0, a_1, a_2, \ldots, a_{n_0-1}, M\}$"

... then your proof is incorrect. **You need to fix a value of $M$ first.** Otherwise your variable $M$ is quantified – it is a dummy variable, and it does not mean anything. Moreover, the value of $n_0$ depends on $M$. The above set does not make sense (or is not finite) unless we fix one single value of $M$ and as a consequence one single value of $n_0$.

- Do not confuse “divergent” with “divergent to $\infty$”. They are very different concepts.