MAT137 - Calculus with proofs

- Test 3 opens on February 5.

Unit 9: Integration methods

- TODAY: Substitution or Chain Rule
  (Videos 9.1, 9.3; Supplementary: 9.2)
- NEXT: Parts or Product Rule
  (**Video: 9.4**; Supplementary: 9.5, 9.6)
- WEDNESDAY: Products of trig functions
  (Video: 9.7; Supplementary: 9.8, 9.9)
- FRIDAY: Rational functions
  (Video: 9.10; Supplementary: 9.11, 9.12)

Computation practice: integration by substitution

Use substitutions to compute:

1. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
2. $\displaystyle\int e^x \cos\left(e^x\right) dx$
3. $\displaystyle\int \cot x\, dx$
4. $\displaystyle\int x^2\sqrt{x+1}\, dx$
5. $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}}\, dx$
6. $\displaystyle\int \frac{(\ln\ln x)^2}{x\ln x}\, dx$
7. $\displaystyle\int xe^{-x^2}\, dx$
8. $\displaystyle\int e^{-x^2}\, dx$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Wrong answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx.$

$$
\begin{aligned}
I &= \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2 dx) & &= \frac{1}{3}\int_0^2 u^{1/2}\, du \\
&= \frac{1}{3}\,\frac{2}{3}\, u^{3/2}\Big|_0^2 & &= \frac{1}{9}\left(x^3+1\right)^{2/3}\Big|_0^2 \\
&= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\left(0+1\right)^{3/2} & &= \frac{52}{9}
\end{aligned}
$$

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

1. Attempt the substitution $u = \sin x$
2. Attempt the substitution $u = \cos x$
3. One worked better than the other. Which one? Why? Finish the problem.
4. Assume we want to compute

$$\int \sin^n x \cos^m x \, dx$$

When will the substitution $u = \sin x$ be helpful?
When will the substitution $u = \cos x$ be helpful?