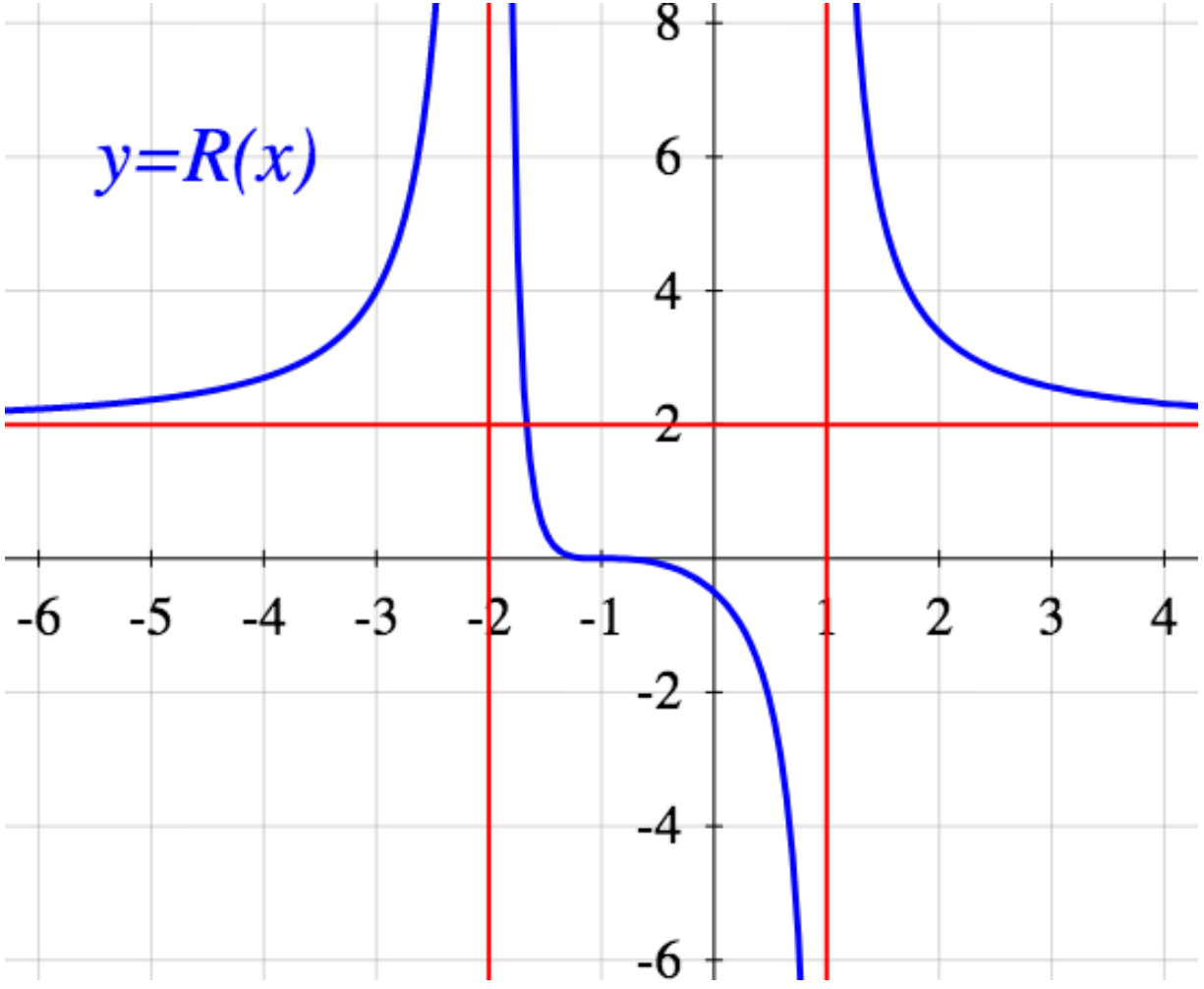
MAT137 - Calculus with proofs

- Assignment #5 due on December 20

- Watch videos 7.1, 7.2 by Monday, January 11. Happy holidays!

Backwards graphing

R is a rational function (a quotient of polynomials).
Find its equation.



Suggestion: Play with desmos.

A very hard function to graph

The function $G(x) = xe^{1/x}$ is deceiving. To help you out:

$$G'(x) = \frac{x-1}{x}e^{1/x}, \qquad G''(x) = \frac{e^{1/x}}{x^3}$$

1. Carefully study the behaviour as $x \to 0^+$ and $x \to 0^-$. The two are very different.
2. Carefully study the behaviour as $x \to \pm\infty$.
   You should find an asymptote, but it is not easy.
3. Use G' to study monotonocity.
4. Use G'' to study concavity.
5. Sketch the graph of G.

Come to the dark side

Help me write a difficult question for Test 3! I will ask you to compute a limit like this

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x + \ ???}{x^?}$$

I have not decided yet what to put instead of the question marks, but I do not want it to look too scary.

I want the calculation to require 6 iterations of L'Hôpital's Rule.

I do not want the answer to be 0 or ∞ or $-\infty$ or "DNE", because you could guess that answer.

What limit should I ask? And what will the answer be?

A polynomial from 3 points

Construct a polynomial that satisfies the following three properties at once:

1. It has an inflection point at $x = 2$
2. It has a a local extremum at $x = 1$
3. It has y-intercept at $y = 1$.

Fractional exponents

Let $h(x) = \dfrac{x^{2/3}}{(x-1)^{2/3}}$. Its first two derviatives are

$$h'(x) = \frac{-2}{3x^{1/3}(x-1)^{5/3}} \qquad h''(x) = \frac{2(6x-1)}{9x^{4/3}(x-1)^{8/3}}$$

1. Find all asymptotes of h
2. Study the monotonicity of h and local extrema
3. Study the concavity of h and inflection points
4. With this information, sketch the graph of h