- Assignment #5 due on December 20

- TODAY: Concavity

- WEDNESDAY: Asymptotes
  - **Watch videos 6.15, 6.16, 6.17**
  - Supplementary video: 6.18
- THURSDAY: Curve sketching (no videos)

Critique this solution:

- Let f be a function with domain $\mathbb{R}$.
- Assume that $f(0) = 0$ and that f is differentiable at 0.
- Calculate $\lim_{x \to 0} \frac{f(x)}{\sqrt[3]{x}}$.

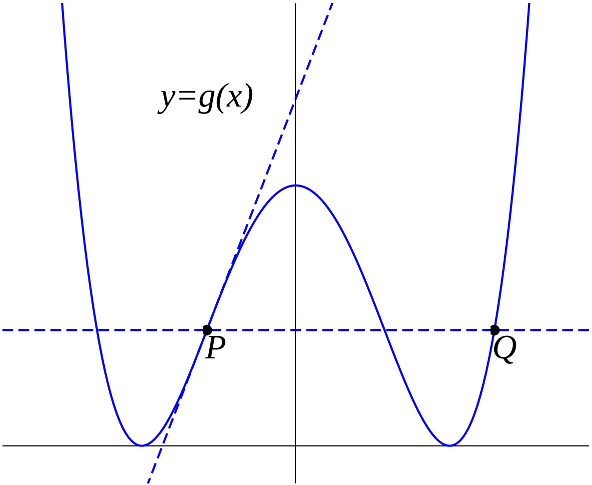
"Solution"

It is an indeterminate form 0/0, so I use L'Hôpital's Rule:

$$\begin{aligned} \lim_{x \to 0} \frac{f(x)}{\sqrt[3]{x}} &= \lim_{x \to 0} \frac{f'(x)}{\frac{1}{3} x^{-2/3}} \\ &= \lim_{x \to 0} \left[3x^{2/3} f'(x) \right] \\ &= 3 \cdot 0 \cdot f'(0) = 0 \end{aligned}$$

Find the coordinates of P and Q

$$g(x) = x^4 - 6x^2 + 9$$



True or False – Concavity and inflection points

Let f be a differentiable function with domain $\mathbb{R}$.
Let $c \in \mathbb{R}$. Let I be an interval. Which implications are true?

1. IF f is concave up on I, THEN $\forall x \in I$, $f''(x) > 0$.
2. IF $\forall x \in I$, $f''(x) > 0$, THEN f is concave up on I.
3. IF f is concave up on I THEN f' is increasing on I.
4. IF f' is increasing on I, THEN f is concave up on I.

5. IF f has an I.P. at c, THEN $f''(c) = 0$.
6. IF $f''(c) = 0$, THEN f has an I.P. at c.
7. IF f has an I.P. at c, THEN f' has a local extremum at c
8. IF f' has a local extremum at c, THEN f has an I.P. at c.

I.P. = "inflection point"

Monotonicity and concavity

Let $f(x) = xe^{-x^2/2}$.

1. Find the intervals where f is increasing or decreasing, and its local extrema.
2. Find the intervals where f is concave up or concave down, and its inflection points.
3. Calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
4. Using this information, sketch the graph of f.