MAT137 - Calculus with proofs

- Assignment #4 due on November 26
- Test 2 opens on December 4
- Assignment #5 due on December 20

- TODAY: Monotonicity

- FRIDAY: Related Rates **(Videos 6.1, 6.2)**
- MONDAY: Optimization (Videos 6.3, 6.4)

Definition of increasing

Let f be the function defined by $f(x) = x^3$.
Which ones of these statements are TRUE?

1. f is increasing on $(0, \infty)$.
2. f is increasing on $[0, \infty)$.
3. f is increasing on $(-\infty, 0)$.
4. f is increasing on $(-\infty, 0]$.
5. f is increasing on $(-\infty, 0)$ and on $(0, \infty)$.
6. f is increasing on $(-\infty, 0]$ and on $[0, \infty)$.
7. f is increasing on $\mathbb{R}$.
8. f is increasing on $[1, 2]$.

True or False – AGAIN

Let I be an OPEN interval.
Let f be a function defined on I.
Let $c \in I$. Which implications are true?

1. IF f is increasing on I, THEN $\forall x \in I,\ f'(x) > 0$.
2. IF $\forall x \in I,\ f'(x) > 0$, THEN f is increasing on I.

3. IF f has a local extremum at c, THEN $f'(c) = 0$.
4. IF $f'(c) = 0$, THEN f has a local extremum at c.

Preparation

1. Let f be a function defined on an interval I.
   Write the definition of "f is increasing on I".

2. Write the statement of the Mean Value Theorem.

Positive derivative implies increasing

Use the MVT to prove

Theorem

Let $a < b$. Let f be a differentiable function on (a, b).

- IF $\forall x \in (a, b), f'(x) > 0$,
- THEN f is increasing on (a, b).

1. Recall the definition of what you are trying to prove.
2. **From that definition, figure out the structure of the proof.**
3. If you have used a theorem, did you verify the hypotheses?
4. Are there words in your proof, or just equations?

What is wrong with this proof?

Theorem

Let $a < b$. Let f be a differentiable function on (a, b).

- IF $\forall x \in (a, b), f'(x) > 0$,
- THEN f is increasing on (a, b).

Proof.

- From the MVT, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$
- We know $b - a > 0$ and $f'(c) > 0$
- Therefore $f(b) - f(a) > 0$. Thus $f(b) > f(a)$.
- f is increasing.

$\square$

Inequalities

Prove that, for every $x \in \mathbb{R}$, $\quad e^x \geq 1 + x$.

Hint: Where is the function $f(x) = e^x - 1 - x$ increasing or decreasing? What is its minimum?