MAT137 - Calculus with proofs

- Assignment #4 due on November 26
- Test 2 opens on December 4
- Assignment #5 due on December 20

- TODAY: The Mean Value Theorem

- WEDNESDAY: Monotonicity
  - **Required videos: 5.10, 5.11**
  - Supplementary video: 5.12

True or False – Local extrema: REVENGE!

Let I be an OPEN interval.
Let f be a DIFFERENTIABLE function defined on I.
Let $c \in I$.
Which implications are true?

1. IF f has local extremum at c, THEN f has an extremum at c
2. IF f has an extremum at c, THEN f has local extremum at c

3. IF f has a local extremum at c, THEN $f'(c) = 0$.
4. IF $f'(c) = 0$, THEN f has a local extremum at c.

Proving difficult identities

Prove that, for every $x \geq 0$,

$$2\arctan\sqrt{x} \; - \; \arcsin\frac{x-1}{x+1} \; = \; \frac{\pi}{2}$$

Hint: You are trying to prove a function is constant. Use derivatives.

Critique this "proof"

- $\left[2\arctan\sqrt{x} - \arcsin\frac{x-1}{x+1}\right] = \left[\frac{\pi}{2}\right]$
- $\frac{d}{dx}\left[2\arctan\sqrt{x} - \arcsin\frac{x-1}{x+1}\right] = \frac{d}{dx}\left[\frac{\pi}{2}\right]$
- $\frac{2}{1+\left(\sqrt{x}\right)^2}\cdot\frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2} = 0$
- $\frac{1}{(1+x)\sqrt{x}} - \frac{1}{\sqrt{\frac{4x}{(x+1)^2}}}\cdot\frac{2}{(x+1)^2} = 0$
- $0 = 0$
- So $2\arctan\sqrt{x} - \arcsin\frac{x-1}{x+1}$ is constant.
- Evaluate at $x = 0$ to find the value of the constant.
- $2\arctan 0 - \arcsin(-1) = 0 - (-\pi/2) = \pi/2$
- Therefore, $2\arctan\sqrt{x} - \arcsin\frac{x-1}{x+1} = \frac{\pi}{2}$

Car race - 1

A driver competes in a race.

Use MVT to prove that at some point during the race the instantaneous velocity of the driver is exactly equal to the average velocity of the driver during the race.

Car race - 2

Two drivers start a race at the same moment and finish in a tie.

Can you conclude that there was a time in the race (not counting the starting time) when the two drivers had exactly the same speed?

Car race - Is this proof correct?

Claim

IF two drivers start a race at the same moment and finish in a tie, THEN at some point in the race (not counting the starting time) they had exactly the same speed.

Proof?

- Let $f(t)$ and $g(t)$ be the positions of the two cars at time t.
- Assume the race happens in the interval $[t_1, t_2]$. By hypothesis:

$$f(t_1) = g(t_1), \qquad f(t_2) = g(t_2).$$

- Using MVT, there exists $\quad c \in (t_1, t_2) \quad$ such that

$$f'(c) = \frac{f(t_2) - f(t_1)}{t_2 - t_1}, \quad g'(c) = \frac{g(t_2) - g(t_1)}{t_2 - t_1}.$$

- Then $f'(c) = g'(c)$. □

Car race - resolution

Two drivers start a race at the same moment and finish in a tie.

Prove that at some point during the race (not counting the starting time) the two drivers had exactly the same speed.