MAT137 - Calculus with proofs

- Assignment #3 due on November 5
- TODAY: Chain Rule
- FRIDAY: Trig and implicit diferentiation
  (Videos 3.12, 3.13)
- MONDAY: Functions and inverse functions
  (Videos 4.1, 4.2)

Quick composition

Let f and g be differentiable functions and let $h = f \circ g$. What is $h'(2)$?

1. $f'(2) \circ g'(2)$
2. $f'(2)g'(2)$
3. $f'(g(2)) \circ g'(2)$
4. $f'(g(2))g'(2)$
5. $f' \circ g'(2)$
6. $f'(g'(2))$
7. $f'(g(x))g'(2)$
8. $f'(g(x)) \circ g'(2)$

True or False - Differentiability and Composition

Let f and g be functions with domain $\mathbb{R}$. Let $c \in \mathbb{R}$.
Assume f and g are differentiable at c.
What can we conclude?

1. $f \circ g$ is differentiable at c.

2. $f \circ f$ is differentiable at c.

3. $f \circ \sin$ is differentiable at c.

4. $\sin \circ f$ is differentiable at c.

Computations: Chain rule

Compute the derivative of

1. $f(x) = \left(2x^2 + x + 1\right)^8$
2. $f(x) = \dfrac{1}{\left(x + \sqrt{x^2 + x}\right)^{137}}$

Estimations – 4

Without using a calculator, estimate $\sqrt[20]{1.01}$.

Hint: You know the value of $f(x) = \sqrt[20]{x}$ and its derivative at one point very close to 1.01. Use the tangent line at that point as an approximation.

Estimations – 5

Let f and g be continuous function with domain $\mathbb{R}$.

We know $\quad f(0) = 0, \quad g(0) = 0,$

Estimate $\lim_{x \to 0} \dfrac{f(x)}{g(x)}$

Estimations – 6

Let f and g be continuous function with domain $\mathbb{R}$.

We know $\quad f(0) = 0, \quad g(0) = 0, \quad f'(0) = 3, \quad g'(0) = 5.$

- When x is close to 0, give estimates for $f(x)$ and $g(x)$ using the tangent lines at 0.
- Use those estimates to "compute" $\displaystyle \lim_{x\to 0} \frac{f(x)}{g(x)}$.

Derivatives of $(f \circ g)$

Assume f and g are functions that have all their derivatives. Find formulas for

1. $(f \circ g)'(x)$
2. $(f \circ g)''(x)$
3. $(f \circ g)'''(x)$

in terms of the values of f, g and their derivatives.

Hint: The first one is simply the chain rule.

Challenge: Find a formula for $(f \circ g)^{(n)}(x)$
(This is beyond the scope of this course).