MAT137 - Calculus with proofs

- Assignment #3 due on November 5
- TODAY: More on differentiation rules
- WEDNESDAY: Chain Rule **(Videos 3.10, 3.11)**
- FRIDAY: Trig and implicit diferentiation (Videos 3.12, 3.13)

True or False - Differentiability vs Continuity

Let f be a function with domain $\mathbb{R}$. Let $c \in \mathbb{R}$.
Which of these implications are true?

1. IF f is continuous at c, THEN f is differentiable at c
2. IF f is differentiable at c, THEN f is continuous at c
3. IF f is differentiable at c, THEN f' is continuous at c
4. IF f' is continuous at c, THEN f is continuous at c
5. IF f is differentiable at c, THEN f is continuous at and near c.
6. IF f is continuous at and near c, THEN f is differentiable at c.

True or False - Differentiability and Operations

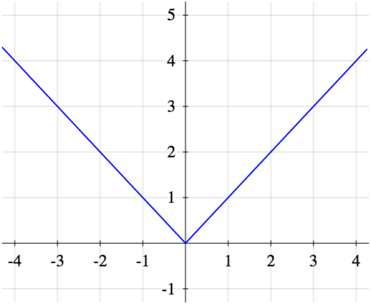
Let f be a function with domain $\mathbb{R}$. Let $c \in \mathbb{R}$.
Let $g(x) = f(x)^2$. Which of these implications are true?

1. IF f is differentiable at c, THEN $3f$ is differentiable at c.
2. IF f is differentiable at c, THEN g is differentiable at c.
3. IF g is differentiable at c, THEN f is differentiable at c.
4. IF f is differentiable at c, THEN $f + g$ is differentiable at c.
5. IF f is differentiable at c, THEN $1/f$ is differentiable at c.

Absolute value and tangent lines

At (0,0) the graph of $y = |x|$...

1. ... has one tangent line: $y = 0$
2. ... has one tangent line: $x = 0$
3. ... has two tangent lines $y = x$ and $y = -x$
4. ... has no tangent line



Absolute value and derivatives

Let $h(x) = x|x|$. What is $h'(0)$?

1. It is 0.
2. It doesn't exist because $|x|$ is not differentiable at 0.
3. It doesn't exist because the right- and left-limits, when computing the derivative, are different.
4. It doesn't exist because it has a corner.
5. All of (2), (3), (4) are true.
6. It doesn't exist for a different reason.

Write a proof for the quotient rule for derivatives

Theorem

- Let $a \in \mathbb{R}$.
- Let f and g be functions defined at and near a. Assume $g(x) \neq 0$ for x close to a.
- We define the function h by $h(x) = \dfrac{f(x)}{g(x)}$.

IF f and g are differentiable at a,
THEN h is differentiable at a, and

$$h'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{g(a)^2}.$$

Write a proof directly from the definition of derivative.
Hint: Imitate the proof of the product rule in Video 3.6.

Check your proof

1. Did you use the *definition* of derivative?
2. Are there words or only equations?
3. Does every step follow logically?
4. Did you only assume things you could assume?

5. Did you assume at some point that a function was differentiable? If so, did you justify it?
6. Did you assume at some point that a function was continuous? If so, did you justify it?

If you answered "no" to Q6, you probably missed something important.

Critique this proof

$$
\begin{aligned}
h'(a) &= \lim_{x\to a} \frac{h(x)-h(a)}{x-a} = \lim_{x\to a} \frac{\dfrac{f(x)}{g(x)} - \dfrac{f(a)}{g(a)}}{x-a} \\
&= \lim_{x\to a} \frac{f(x)g(a)-f(a)g(x)}{g(x)g(a)\,(x-a)} \\
&= \lim_{x\to a} \frac{f(x)g(a)-f(a)g(a)+f(a)g(a)-f(a)g(x)}{g(x)g(a)\,(x-a)} \\
&= \lim_{x\to a} \left\{ \left[\frac{f(x)-f(a)}{x-a} g(a) - f(a) \frac{g(x)-g(a)}{x-a} \right] \frac{1}{g(x)g(a)} \right\} \\
&= \left[f'(a)g(a) - f(a)g'(a) \right] \frac{1}{g(a)g(a)}
\end{aligned}
$$