# MAT137 - Calculus with proofs

- Assignment #2 due on Thursday.
- Practice Test: Friday 3pm to Saturday 3pm

- TODAY: More continuity

- FRIDAY: Limit computations! **(Videos 2.19, 2.20)**

# What can we conclude?

Let $c \in \mathbb{R}$. Let $f$ and $g$ be functions.
Assume $f$ and $g$ have removable discontinuities at $c$.
What can we conclude about $f + g$ at $c$?

1. $f + g$ must have a discontinuity at $c$
2. $f + g$ may have a discontinuity at $c$
3. $f + g$ must have a removable discontinuity at $c$
4. $f + g$ may have a removable discontinuity at $c$
5. $f + g$ must have a non-removable discontinuity at $c$
6. $f + g$ may have a non-removable discontinuity at $c$

# Which one is the correct claim?

## Claim 1?

(Assuming these limits exist)

$$\lim_{x \to a} g(f(x)) \;=\; g\left(\lim_{x \to a} f(x)\right)$$

## Claim 2?

IF (A) $\lim_{x \to a} f(x) = L$, and (B) $\lim_{t \to L} g(t) = M$

THEN (C) $\lim_{x \to a} g(f(x)) = M$

# Fix it!

## This claim is false

IF (A) $\lim_{x\to a} f(x) = L$, and (B) $\lim_{t\to L} g(t) = M$

THEN (C) $\lim_{x\to a} g(f(x)) = M$

Which additional hypotheses would make it true?

1. $f$ is continuous at $a$
2. $g$ is continuous at $L$
3. IF $x$ is near $a$ (but $x \neq a$), THEN $f(x) \neq L$
4. IF $t$ is near $L$ (but $t \neq L$), THEN $g(t) \neq M$

# A difficult example

Construct a pair of functions $f$ and $g$ such that

$$\begin{aligned} \lim_{x\to 0} f(x) &= 1 \\ \lim_{t\to 1} g(t) &= 2 \\ \lim_{x\to 0} g(f(x)) &= 42 \end{aligned}$$

# Continuity and quantifiers

Let $f$ be a function with domain $\mathbb{R}$.
Which statements are equivalent to "$f$ is continuous"?

1. $$\forall \varepsilon > 0,\ \exists \delta > 0,\ \boxed{\forall x \in \mathbb{R}},\\ |x - a| < \delta \implies |f(x) - f(a)| < \varepsilon$$

2. $$\boxed{\forall a \in \mathbb{R}},\ \forall \varepsilon > 0,\ \exists \delta > 0,\ \boxed{\forall x \in \mathbb{R}},\\ |x - a| < \delta \implies |f(x) - f(a)| < \varepsilon$$

3. $$\boxed{\forall x \in \mathbb{R}},\ \forall \varepsilon > 0,\ \exists \delta > 0,\ \boxed{\forall a \in \mathbb{R}},\\ |x - a| < \delta \implies |f(x) - f(a)| < \varepsilon$$

4. $$\forall \varepsilon > 0,\ \exists \delta > 0,\ \boxed{\forall a \in \mathbb{R}},\ \boxed{\forall x \in \mathbb{R}},\\ |x - a| < \delta \implies |f(x) - f(a)| < \varepsilon$$