- Test 4 opens on March 12
- Assignment 9 due on March 25

- Today: Properties of series

- Friday: More properties of series
  Watch videos 13.8, 13.9

Geometric series

Calculate the value of the following series:

1. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$
2. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \dots$
3. $\frac{3}{2} - \frac{9}{4} + \frac{27}{8} - \frac{81}{16} + \dots$
4. $1 + \frac{1}{2^{0.5}} + \frac{1}{2} + \frac{1}{2^{1.5}} + \frac{1}{2^2} + \frac{1}{2^{2.5}} + \dots$
5. $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{2^{2n+1}}$
6. $\sum_{n=k}^{\infty} x^n$

Is $0.999999\ldots = 1$?

We can interpret any finite decimal expansion as a finite sum. For example:

$$2.13096 = 2 + \frac{1}{10} + \frac{3}{10^2} + \frac{0}{10^3} + \frac{9}{10^4} + \frac{6}{10^5}$$

Similarly, we can interpret any infinite decimal expansion as an infinite series.

1. Write the number $0.9999999\ldots$ as a series

2. Add up the series.
   Hint: it is geometric.

Examples

1. A series $\sum_{n=0}^{\infty} a_n$ may be

$$\begin{cases} \text{convergent (a number)} \\ \text{divergent} \begin{cases} \text{to } \infty \\ \text{to } -\infty \\ \text{``oscillating''} \end{cases} \end{cases}$$

   Give one example of each of the four results.

2. Now assume $\forall n \in \mathbb{N},\ a_n \geq 0$.
   Which of the four outcomes is still possible?

True or False – Definition of series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the the series $\sum_{n=0}^{\infty} a_n$ is convergent,
   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded.

2. IF the series $\sum_{n=0}^{\infty} a_n$ is convergent,
   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.

3. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic,
   THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

True or False – Definition of series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

4. IF $\forall n > 0,\ a_n > 0$,
   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing.

5. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing,
   THEN $\forall n > 0,\ a_n > 0$.

6. IF $\forall n > 0,\ a_n \geq 0$,
   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is non-decreasing.

7. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is non-decreasing,
   THEN $\forall n > 0,\ a_n \geq 0$