- Assignment 8 due on March 4
- Test 4 opens on March 12

- Today: Basic Comparison Test

- Friday: Limit Comparison Test
  (Watch videos 12.9, 12.10)

Quick review

For which values of $p \in \mathbb{R}$ is each of the following improper integrals convergent?

1. $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$

2. $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$

3. $\displaystyle\int_0^\infty \frac{1}{x^p}\,dx$

A simple BCT application

We want to determine whether $\displaystyle\int_1^\infty \frac{1}{x+e^x}\,dx$ is convergent or divergent.

We can try at least two comparisons:

1. Compare $\dfrac{1}{x}$ and $\dfrac{1}{x+e^x}$.
2. Compare $\dfrac{1}{e^x}$ and $\dfrac{1}{x+e^x}$.

Try both. What can you conclude from each one of them?

True or False - Comparisons

Let $a \in \mathbb{R}$.
Let f and g be continuous functions on $[a, \infty)$.

Assume that $\boxed{\forall x \geq a, \quad 0 \leq f(x) \leq g(x)}$.

What can we conclude?

1. IF $\int_a^\infty f(x)dx$ is convergent, THEN $\int_a^\infty g(x)dx$ is convergent.
2. IF $\int_a^\infty f(x)dx = \infty$, THEN $\int_a^\infty g(x)dx = \infty$.
3. IF $\int_a^\infty g(x)dx$ is convergent, THEN $\int_a^\infty f(x)dx$ is convergent.
4. IF $\int_a^\infty g(x)dx = \infty$, THEN $\int_a^\infty f(x)dx = \infty$.

True or False - Comparisons II

Let $a \in \mathbb{R}$.
Let f and g be continuous functions on $[a, \infty)$.

Assume that $\boxed{\forall x \geq a, \quad f(x) \leq g(x)}$.

What can we conclude?

1. IF $\int_a^\infty f(x)dx$ is convergent, THEN $\int_a^\infty g(x)dx$ is convergent.
2. IF $\int_a^\infty f(x)dx = \infty$, THEN $\int_a^\infty g(x)dx = \infty$.
3. IF $\int_a^\infty g(x)dx$ is convergent, THEN $\int_a^\infty f(x)dx$ is convergent.
4. IF $\int_a^\infty g(x)dx = \infty$, THEN $\int_a^\infty f(x)dx = \infty$.

True or False - Comparisons III

Let $a \in \mathbb{R}$.
Let f and g be continuous functions on $[a, \infty)$.
Assume that $\boxed{\exists M \geq a \text{ s.t. } \forall x \geq M, \quad 0 \leq f(x) \leq g(x)}$.
What can we conclude?

1. IF $\int_a^\infty f(x)dx$ is convergent, THEN $\int_a^\infty g(x)dx$ is convergent.
2. IF $\int_a^\infty f(x)dx = \infty$, THEN $\int_a^\infty g(x)dx = \infty$.
3. IF $\int_a^\infty g(x)dx$ is convergent, THEN $\int_a^\infty f(x)dx$ is convergent.
4. IF $\int_a^\infty g(x)dx = \infty$, THEN $\int_a^\infty f(x)dx = \infty$.

BCT calculations

Use BCT to determine whether each of the following is convergent or divergent

1. $\displaystyle\int_1^\infty \frac{1+\cos^2 x}{x^{2/3}}\,dx$
2. $\displaystyle\int_1^\infty \frac{1+\cos^2 x}{x^{4/3}}\,dx$
3. $\displaystyle\int_0^\infty \frac{\arctan x^2}{1+e^x}\,dx$
4. $\displaystyle\int_0^\infty e^{-x^2}\,dx$
5. $\displaystyle\int_2^\infty \frac{(\ln x)^{10}}{x^2}\,dx$