MAT137 - Calculus with proofs

- Assignment 7 due on February 25
- Assignment 8 due on March 4
- Test 4 opens on March 12

- Today: Properties of sequences
- Wednesday: Proving theorems about sequences (**Watch Videos 11.5, 11.6**)

Sequences vs functions – monotonicity and boundness

For any function f with domain $[0, \infty)$,
we define a sequence as $a_n = f(n)$.
Which of these implications is true?

1. IF f is increasing, THEN $\{a_n\}_{n=0}^{\infty}$ is increasing.
2. IF $\{a_n\}_{n=0}^{\infty}$ is increasing, THEN f is increasing.
3. IF f is bounded, THEN $\{a_n\}_{n=0}^{\infty}$ is bounded.
4. IF $\{a_n\}_{n=0}^{\infty}$ is bounded, THEN f is bounded.

Examples

Construct 8 examples of sequences.
If any of them is impossible, cite a theorem to justify it.

		convergent	divergent
monotonic	bounded		
	unbounded		
not monotonic	bounded		
	unbounded		

True or False - convergence, monotonicity, and boundedness

1. If a sequence is convergent, then it is bounded above.
2. If a sequence is bounded, then it is convergent
3. If a sequence is convergent, then it is eventually monotonic.
4. If a sequence is positive and converges to 0, then it is eventually monotonic.
5. If a sequence diverges to ∞, then it is eventually monotonic.
6. If a sequence diverges, then it is unbounded.
7. If a sequence diverges and is unbounded above, then it diverges to ∞.
8. If a sequence is eventually monotonic, then it is either convergent, divergent to ∞, or divergent to $-\infty$.

Convergence and divergence

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence. Write the formal definition of:

1. $\{a_n\}_{n=0}^{\infty}$ is convergent.

2. $\{a_n\}_{n=0}^{\infty}$ is divergent.

3. $\{a_n\}_{n=0}^{\infty}$ is divergent to ∞.